GAME THEORY

INTRODUCTION

Competition is a 'Key factor' of modern life. We say that a competitive situation exists if two or more individuals are taking decisions in situation that involves conflicting interests and in which the outcome is controlled by the decisions of all parties concerned. We assume that in a competitive situation each participant acts in a rational manner and tries to resolve the conflicts of interests in his favour. It is in this context that game theory has developed. Professor John von Newmann and Oscar Morgensten published their book entitled "The Theory of Games and Economic Behaviour" where in they provided a new approach to many problems involving conflict situations. This approach is now widely used in Economics, Business Administration, Sociology, Psychology and Political Science as well as in Military Training. In games like chess, draught, pocker etc. which are played as per certain rules victory of one side and the defeat of the other is dependent upon the decisions based in skillful evaluation of the alternatives of the opponent and also upon the selection of the right alternative.

Game Theory is a body of knowledge which is concerned with the study of decision making in situation where two or more rational opponents are involved under condition of competition and conflicting interests. It deals with human processes in which an individua1 decision making unit is not in complete control of the other decision making units. Here ε unit may be an individual group, organisation, society or a country.

Game Theory is a type of decision theory which is based on reasoning in which the choice of action is determined after considering the possible alternatives available to the opponents playing the same game. The aim is to choose the best course of action, because every player has got an alternative course of action.

Significance of Game Theory

We know that competition is a major factor in modern life. There are so many competition like business competition, elections competitions, sport competition etc. In all these competition persons have conflicting interest, and every body tries to maximise his gains and minimise loss.

Game theory is a type of decision theory which is based on the choice of action. A choice of action is determined after considering the possible alternatives available to the opponent. It involves the players i.e. decision makers who have different goals or objectives. The game theory determine the rules of rational behaviour of these players in which outcomes are dependent on the actions of the interdependent players.

In a game there are number of possible outcomes, with different values to the decision makers. They might have some control but do not have the complete control over others example players in a chess game, labour union striking against the management, companies striving for larger share of market etc are the situations where theory of games is applicable because each situation can be viewed as games. So, game theory is important weapon in the hands of management. Game theory is a scientific approach rational decision making.

Essential features of Game Theory

A competitive situation is called a game if it has the following features:

i. **Finite Number of Competitors**. There are finite number of competitors, called players. The players need not be individuals, they can be groups, corporations, political parties, institutions or even nations.
ii. **Finite Number of Action.** A list of finite number of possible courses of action is available to each player. The list need not be the same for each player.
iii. **Knowledge of Alternatives.** Each player has the knowledge of alternatives available to his opponent.
iv. **Choice.** Each player makes a choice, i.e., the game is played. The choices are assumed to be made simultaneously, so that no player knows his opponents' choice until he has decided his own course of action.
v. **Outcome or Gain**. The play is associated with an outcome known as gain. Here the loss is considered negative gain.
vi. **Choice of Opponent.** The possible gain or loss of each player depends upon not only the choice made by him but also the choice made by his opponent.

Two Persons Zero-sum Game. Two person zero-sum game is the situation which involves two persons or players and gains made by one person is equals to the loss incurred by the other. For example there are two companies Coca-cola & Pepsi and are struggling for a larger share in the market. Now any share of the market gained by the Coca-cola company must be the lost share of Pepsi, and therefore, the sums of the gains and losses equals zero. In other words if gain of Coca-cola is 40% so the lost share of the Pepsi will be 40%. The sum of game and losses is zero i.e. (+40%)+(-40%) = 0

n-persons game. A game involving n persons is called a n persons game. In this two persons game are most common. When there are more than two players in a game, obviously the complexity of the situation is increased.

Pay offs. Outcomes of a game due to adopting the different courses of actions by the competing players in the form of gains or losses for each of the players is known as pay offs.

Payoff Matrix. In a game, the gains and losses, resulting from different moves and counter moves, when represented in the form of a matrix are known as payoff matrix or gain matrix. This matrix shows how much payment is to be made or received at the end of the game in case a particular strategy is adopted by a player.

Payoff matrix shows the gains and losses of one of the two players, who is indicated on the left hand side of the pay off matrix. Negative entries in the matrix indicate losses. This is generally prepared for the maximising player. However the same matrix can be interpreted for the other player also, as in a zero sum game, the gains of one player represent the losses of the other player, and *vice versa.* Thus, the payoff matrix of Mr A is the negative payoff matrix for Mr B. The other player is known as the minimising player. He is indicated on the top of the table.

The payoff matrix can be understood by the following example.

		Y	
		Y_1	Y_2
X	X_1	6	9
	X_2	12	15

In our example we have two players X and Y. X has two strategies X_1 and X_2. Similarly Y also has two strategies Y_1 and Y_2.:

X's	:	X_1	X_2	X_1	X_2
Y's,	:	Y_1	Y_1	Y_2	Y_2
Payoff	:	6	12	9	15

The above arrangement of the matrix is known as payoff matrix. It should always be written from the X's point of view *i.e.* from the player who is on the left hand side.

Decision of a Game. In game theory best strategy for each player is determined on the basis of some criteria. Since both the players are expected to be rational in their approach, this is known as the criteria of optimality. Each player lists the possible outcomes from his objective and then selects the best strategy out of these outcomes from his point of view or as per his objective.

The decision criteria in game theory is known as the criteria of optimality, *i.e.,* maximin for the maximising player and minimax for the minimising player.

Limitation of Game Theory

1. **Infinite number of strategy.** In a game theory we assume that there is finite number of possible courses of action available to each player. But in practice a player may have infinite number of strategies or courses of action.
2. **Knowledge about strategy.** Game theory assumes that each player as the knowledge of strategies available to his opponent. But some times knowledge about strategy about the opponent is not available to players. This leads to the wrong conclusions.

Zero outcomes. We have assumed that gain of one person is the loss of another person. But in practice gain of one person may not be equal to the loss of another person i.e. opponent.

Risk and uncertainty. Game theory does not takes into consideration the concept of probability. So game theory usually ignores the presence of risk and uncertainty.

Finite number of competitors. There are finite number of competitors as has been assumed in the game theory. But in real practice there can be more than the expected number of players.

Certainity of Pay off. Game theory assume that payoff is always known in advance. But sometimes it is impossible to know the pay off in advance. The decision situation infact becomes multidimensional with large number of variables.

Rules of Game. Every game is played according to the set of rules i.e specific rules which governs the behaviour of the players. *As* there we have set of rules of playing Chess, Badminton, Hockey etc.

Strategy. It is the pre-determined rule by which each player decides his course of action from his list available to him. How one course of action is selected out of various courses available to him is known as strategy of the game.

Types of Strategy. Generally two types of strategy are employed

(i) Pure Strategy (ii) Mixed Strategy

(i.) Pure Strategy. It is the predetermined course of action to be employed by the player. The players knew it in advance. It is usually represented by a number with which the course of action is associated.

(ii.) Mixed Strategy. In mixed strategy the player decides his course of action in accordance with some fixed probability distribution. Probability are associated with each course of action and the selection is done as per these probabilities.

In mixed strategy the opponent can not be sure of the course of action to be taken on any particular occasion.

Decision of a Game. In Game theory, best strategy for each player is determined on the basis of some rule. Since both the players are expected to be rational in their approach this is known as the criteria of optimality. Each player lists the possible outcomes from his action and selects the best action to achieve his objectives. This criteria of optimality is expressed as Maximin for the maximising player and Minimax for the minimising player.

THE MAXIMIN-MINIMAX PRINCIPLE

(i) **Maximin Criteria:** The maximising player lists his minimum gains from each strategy and selects the strategy which gives the maximum out of these minimum gains.

(ii) **Minimax Criteria A** : The minimising player lists his maximum loss from each strategy and selects the strategy which gives him the minimum loss out of these maximum losses.

For Example Consider a two person zero sum game involving the set of pure strategy for Maximising player A say A_1 A_2 & As and for player B, B_1 & B_2, with the following payoff

Player B

		B_1	B_2	Row minima
Player A	A_1	9	2	2
	A_2	8	6	6 * Maximin
	A_3	6	4	4
Column Maxima		9	6 * Minimax	

Since Maximin = Minimax

V = 6

Suppose that player A starts the game knowing fully well that whatever strategy he adopts B will select that particular counter strategy which will minimise the payoff to A. If A selects the strategy Al then B will select B_2 so that A may get minimum gain. Similarly if A chooses A_2 than B will adopt the strategy of B_2. Naturally A would like to maximise his maximin gain which is just the largest of row minima. Which is called 'maximin strategy'. Similarly B will minimise his minimum loss which is called 'minimax strategy'. We observe that in the above example, the maximum if row minima and minimum of column maxima are equal. In symbols.

Maxi [Min.] = Mini [Max]

The strategies followed by both the p1ayersarecalled ‘optimum strategy’.

Value of Game. In game theory, the concept value of game is considered as very important. The value of game is the maximum guaranted gain to the maximising player if both the players use their best strategy. It refers to the average payoff per play of the game over a period of time. Consider the following the games.

$$\text{Player X} \quad \overset{\text{Player Y}}{\begin{bmatrix} 3 & 4 \\ -6 & -2 \end{bmatrix}} \qquad \text{Player X} \quad \overset{\text{Player Y}}{\begin{bmatrix} -7 & 2 \\ -3 & -1 \end{bmatrix}}$$

(with positive value) (with negative value)

In the first game player X wins 3 points and the value of the value is three with positive sign and in the second game the player Y wins 3 points and the value of the game is -ve which indicates that Y is the Winner. The value is denoted by *'v.*

Saddle Point. The Saddle point in a pay off matrix is one which is the smallest value in its row and the largest value in its column. The saddle point is also known as equilibrium point in the theory of games. An element of a matrix that is simultaneously minimum of the row in which it occurs and the maximum of the column in which it occurs is a saddle point of the matrix game. In a game having a saddle point optimum strategy for a player X is always to play the row containing saddle point and for a player Y to play the column that contains saddle point. The following steps are required to find out Saddle point;

(i) Select the minimum value of each row & put a circle ○ around it.

(ii) Select the maximum value of each column and put square □ around it.

(iii) The value with both circle & square is the saddle point.

In case there are more than one Saddle point there exist as many optimum points or solutions of the game. There mayor may not be the saddle point in the game. When there is no saddle point we have to use algebraic methods for working out the solutions concerning the game problem. Saddle point can be found out in different ways.

	Player B
Player A	$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{bmatrix} B_1 & B_2 \\ 20 & 80 \\ 40 & 30 \\ 50 & 60 \end{bmatrix}$

1st Method

Step 1. Putting circle ○ around Row minima.
Step 2. Putting square □ around Column Maxima.
Step 3. Saddle point is a point where circle & square are both combined.

Player B

Player A		B_1	B_2
	A_1	20	$\boxed{80}$
	A_2	40	30
	A_3	$\boxed{50}$	60

Value of game = V = 50

IInd Method

or (i) Putting mark* on each Row minima.
(ii) Putting mark* on each Column Maxima.
(iii) Saddle point where both* and (different star) appears.

Player A		B_1	B_2
	A_1	20*	80*
	A_2	40	30*
	A_3	50**	60

Value of game (υ) = 50

IIIrd Method

OR (i) Creating column for minimum value of each row
(ii) Creating Row for Maximum value of each column.

	B_1	B_2	Row Minima
A_1	20	80	20
A_2	40	30	30
A_3	50	60	50* Maximum
Column Maxima	50* Minimax	80	

The same value in Row minima & Column maxima is the value of game. The optimal strategy for A is A_3 and for B is B_1

(Students can apply any method they like.)

Points to remember

(i) Saddle point mayor may not exist in a given game.

(ii) There may be more than one saddle point then there will be more than one solution. (Such situation is rare in the rea1life).

(iii) The value of game may be +ve or -ve.

(iv) The value of game may be zero which means 'fair game'.

TYPES OF PROBLEMS

(1) GAMES WITH PURE STRATEGIES OR
TWO PERSON ZERO SUM GAME WITH SADDLE POINT,
OR
TWO PERSON ZERO SUM WITH PURE STRATEG'Y.

In case of pure strategy, the maximising player arrives at his optimal strategy on the basis of maximin criterion. The game is solved when maximin value equals minimax value.

Example 2. Solve the following game

		Firm Y		
		Y_1	Y_2	Y_3
Firm X	X_1	4	20	6
	X_2	18	12	10

Solution X is maximising player & Y is minimising player. If Firm X chooses X_1 then Firm Y will choose Y_1 as a counter strategy resulting in payoff equal to 4 to X.

On the other hand if X chooses X_2 then firm Y will choose Y_3 as counter strategy these giving payoff 10 to X. From the firm Y point of view. Strategy Y_3 is better than Y_2. If Y choose Y_1, X will choose X_2 and Y will loose 18 points. On the other hand if Y choose Y_3, firm X will choose X_2 and Y will loose 10 points. So preferred strategy for X is X_2 and for Y is Y_3.

The above problem can be simplified with the help of maximin and minimax criterion as follows :

		Firm Y		
		Y_1	Y_2	Y_3
Firm X	X_1	4	$\boxed{20}$	6
	X_2	$\boxed{18}$	12	$\boxed{10}$

○ = Row Minima
□ = Column Maxima

The saddle point exists and the value of Game *(v)* is 10 and the pure strategy for X is X_2 and for Y is Y_3

II GAMES WITH MIXED STRATEGIES

All game problems, where saddle point does not exist are taken as mixed strategy problems. Where row minima is not equal to column maxima, then different methods are used to solve the different types of problems. Both players will use different strategies with certain probabilities to optimise. For the solution of games with mixed strategies, any of the following methods can be applied.

1. ODDS METHOD
   (2x2 game without saddle point)
2. Dominance Method.
3. Sub Games Method. – For (mx2) or (2xn) Matrices
4. Equal Gains Method.
5. Linear Programming Method-Graphic solution
6. Algebraic method.
7. Linear programming - Simplex method
8. Itirative method

These methods are explained one by one with examples, in detail.

1. ODDS Method - For 2 x 2 Game

Use of odds method is possible only in case of games with 2 x 2 matrix. Here it should be ensured that the sum of column odds and row odds is equal.

METHOD OF FINDING OUT ODDS

Step 1. Find out the difference in the value of in cell (1, 1) and the value in the cell (1,2) of the first row and place it in front of second row.

Step 2. Find out the difference in the value of cell (2, 1) and (2, 2) of the second row and place it in front of first row.

Step 3. Find out the differences in the value of cell (1, 1) and (2, 1) of the first column and place it below the second column.

Step 4. Similarly find the difference between the value of the cell (1, 2) and the value in cell (2, 2) of the second column and place it below the first column.
The above odds or differences are taken as positive (ignoring the negative sign)

Strategy →		Y		
↓		Y_1	Y_2	ODDS
X	X_1	a_1	a_2	→ $(b_1 - b_2)$
	X_2	b_1	b_2	→ $(a_1 - a_2)$
ODDS		↓ $(a_2 - b_2)$	↓ $(a_1 - b_1)$	

The value of game is determined with the help of following equation.

Value of the game (υ) $= \dfrac{a_1(b_1 - b_2) + b_1(a_1 - a_2)}{(b_1 - b_2) + (a_1 - a_2)}$

Probabilities for $X_1 = \dfrac{b_1 - b_2}{(b_1 - b_2) + (a_1 - a_2)}$, $X_2 = \dfrac{a_1 - a_2}{(b_1 - b_2) + (a_1 - a_2)}$

Probabilities for $Y_1 = \dfrac{a_2 - b_2}{(a_2 - b_2) + (a_1 - b_1)}$, $Y_2 = \dfrac{a_1 - b_1}{(a_2 - b_2) + (a_1 - b_1)}$

Example: Solve the following game by odds method.

Strategy		Player B	
		B_1	B_2
Player A	A_1	1	5
	A_2	4	2

Solution

		Player B	
		B_1	B_2
Player A	A_1	1	$\boxed{5}$
	A_2	4	2

Since the game does not have saddle point, the players will use mixed strategy. We apply odds methods to solve the game.

	B_1	B_2	odds
A_1	1	5	4 – 2 = 2
A_2	4	2	1 – 5 = 4
Odds	5 – 2 = 3	1 – 4 = 3	

Value of the game (υ) $= \frac{(1\times2)+(4\times4)}{2+4} = 3, \qquad \upsilon = 3$

Probabilities of Selecting Strategies

		I	II
	A	1/3	2/3
Players	B	1/2	1/2

2. Dominance Method. Dominance method is also applicable to pure strategy and mixed strategy problem. In pure strategy the solution is obtained by itself while in mixed strategy it can be used for simplifying the problem.

Principle of Dominance. The Principle of Dominance states that if the strategy of a player dominates over the other strategy in all condition then the later strategy is ignored because it will not effect the solution in any way. For the gainer point of view if a strategy gives more gain than another strategy, then first strategy dominates over the other and the second strategy can be ignored altogether. Similarly from loser point of view, if a strategy involves lesser loss than other in all condition then second can be ignored. So determination of superior or inferior strategy is based upon the objective of the player. Since each player is to select his best strategy, the inferior strategies can be eliminated. In other words, ineffective rows & column can be deleted from the game matrix and only effective rows & columns of the matrix are retained in the reduced matrix.

For deleting the ineffective rows & columns the following general rules are to be followed.

1. If all the elements of a row (say ith row) ofa pay off matrix are less than or equal to (≤) the corresponding each element of the other row (say jth row) then the player A will never choose the ith strategy OR ith row is dominated by jth row. Then delete ith row.

   Eg. $E_{ij} - [R_{ith}] \leq E_{ij}\begin{pmatrix} R_{ith} \\ Row \end{pmatrix}$ Delete R_{ith} rows.
2. If all the elements of a column (say jth column are greater than or equal to the corresponding elements of any other column (say jth column) then ith column is dominated by jth column. Then delete ith column.

   $E_{ij}(c_i) \geq E_{ij}(cj)$

   delete $= c_i$ th

3. A pure strategy of a player may also be dominated if it is inferior to some convex combination of two or more pure strategies. As a particular case, if all the elements of a column are greater than or equal to the average of two or more other columns then this column is dominated by the group of columns. Similarly if all the elements of row are less than or equal to the average of two or more rows then this row is dominated by other group of row.

4. By eliminating some of the dominated rows a columns and if the game is reduced to 2 x 2 form it can be easily solved by odds method.

Example: Solve the game.

	B		
A	5	20	-10
	10	6	2
	20	15	18

		B		
		I	II	III
A	I	5	20	-10
	II	10	6	2
	III	20	15	18

Since there is no saddle point, so we apply dominance method. Here Row II dominates Row II so we will delete Row II.

		B		
		I	II	III
A	I	5	20	-10
	III	20	15	18

Since column III dominates column I, we delete column I we get:

		B		
		II	III	odds
A	I	20	-10	3
	III	15	18	30
	Odds	28	5	33

$$\text{Value of the game } = \frac{20(3)+15(30)}{3+30} = \frac{510}{33} = \frac{170}{11}$$

Probability of Selecting Strategies

Player A
I = 3 / 33 = 1/ 11
II = 0
III = 30/33 = 10/11

Player B
I = 0
II = 28 / 33
III = 5/33

Using the dominance property obtain the optimal strategies for both the players and determine the value of the game. The pay off matrix for player A is given.

		Player B				
		I	II	III	IV	V
Player A	I	2	4	3	8	4
	II	5	6	3	7	8
	III	6	7	9	8	7
	IV	4	2	8	4	3

Solution.

Since the questions desires to show the dominance property, that is why we are using it, even if the question may have saddle point.

Since all elements in Row IV are less than respective each element in Row III. Row III dominates Row IV. Hence we delete Row IV & we get

		Player B				
		I	II	III	IV	V
Player A	I	2	4	3	8	4
	II	5	6	3	7	8
	III	6	7	9	8	7

Now all the elements of column I are less than or equal to respective each elements of column IV, so we can delete column IV now we get

		Player B			
		I	II	III	V
Player A	I	2	4	3	4
	II	5	6	3	8
	III	6	7	9	7

Repeating the above rules now each element of column I is less than the respective elements of column V, we can delete the column V and we get.

	B		
	I	II	III
I	2	4	3
II	5	6	3
III	6	7	9

Now as each element of Row I and Row II are less than the respective elements of III, we can delete both Rows I & II

		Player B		
		I	II	III
Player A	III	6	7	9

Value of Game (v) = 6

Strategy for player A $\rightarrow$ III

Strategy for player B $\rightarrow$ I

3. LPP – Graphical Method

For (2xm) and (nx2) matrix

Graphic method is applicable to only those games in which one of the players has two strategies only. Through sub-game method provides simple approach, but in case *'n'* or *'m'* is large then a graphic method is easy and relatively fast.

The following are the steps involved in this method.

Step. (i) The game matrices of 2xm or nx2 is divided into 2x2 sub matrices.
Step. (ii) Next taking the probabilities of the two alternatives of the first player say A as P_1 and (1- P_1) the net gain of A from the different alternatives strategies of B is expressed with equation.
Step. (iii) The bounr1aries of the two alternative strategies of the first player we shown by two parallel line shown on the graph.
Step. (iv) The gain equation of different sub games are then plotted on the graph.
Step. (v) In case of maximising player A, the point is identified where minimum expected gain is maximised. This will be the highest point-out the inter section of the gain lines in the 'Lower Envelop'. .

In case of minimising player B, the point where maximum loss is minimised is justified. This will be the lowest point at the intersection of the equations in the 'Upper Envelop'.

Mathematically the method is explained as follows:

Take the example of 2 x *n* game without saddle point with the following pay of matrix.

Player B **Probability**

Player A

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array}\begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & & B_n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots\ldots & a_{2n} \end{bmatrix} \end{array}\begin{array}{c} \\ p_1 \\ 1-p_1 \end{array}$$

Here the objective is to find out the optimum value of p_1 & p_2. For each of the pure strategies B_l, B_2, $B_{3.....}B_{n,}$ available to B the form of equation as followed.

$$B_1 = a_{11}p_1 + a_{21}(1 - p_1) = (a_{11} - a_{21})\, p_1 + a_{21}$$
$$B_2 = a_{12}p_2 + a_{22}(1 - p_1) = (a_{12} - a_{22})\, p_1 + a_{22}$$
$$B_3 = a_{13}p_1 + a_{23}(1 - p_1) = (a_{13} - a_{23})\, p_1 + a_{23}$$
$$\vdots \qquad\qquad\qquad\qquad \vdots$$
$$B_n = a_{1n}p_1 + a_{2n}(1 - p_1) = (a_{1n} - a_{2n})\, p_1 + a_{2n}$$

According to maximin or minimax criteria the player A will select the value of p_1 so to maximize his minimum payoff. This is done by drawing the following straight lines function of x_1

$E_1 = (a_{11} - a_{21})\ x_1 + a_{21}$
$E_2 = (a_{12} - a_{22})\ x_2 + a_{22}$
$E_3 = (a_{13} - a_{23})\ x_3 + a_{23}$
:   :   :
:   :   :
$E_n = (a_{1n} - a_{2n})\ x_n + a_{2n}$

The parallel lines one unit apart representing the two strategies of player A are drawn., For drawing B's strategies lines. We join au on scale 1 to a_{21} on scale 2 for representing B_l corresponding to line $E_1 = (a_{11} - a_{21})\ x_1 + a_{21}$ with x_l as X-axis and E_1 as Y-axis. Similarly other payoff lines can be drawn.

The lower boundary of these lines will give the minimum expected payoff and the highest point on this lower boundary give the maximum expected payoff to A, hence the optimal values of p_1 The two optimum strategies for B are given by the two lines which pass through maximum point.

The m x 2 games are also treated in the same manner except that minimax point is the lowest point on the upper boundary of the straight lines.

Example: Solve the following game graphically.

		1	2	3	4	5
A	1	-5	5	0	-1	8
	2	8	-4	-1	6	-5

Solution

		1	2	3	4	5	Row Minima
A	1	-5	5	0	-1	8	-5* Maximin
	2	8	-4	-1	6	-5	-5*
Column Maxima		8	5	0* Minimax	6	8	

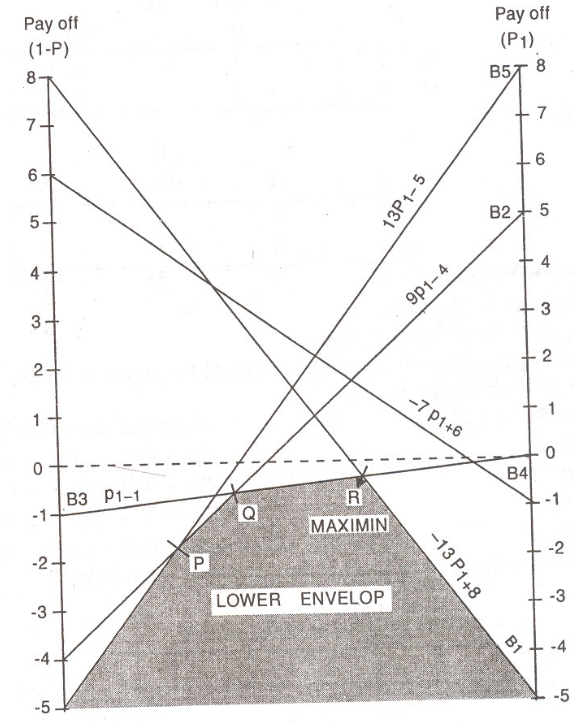
Since, Minimax ≠ Maximin

Thus, players will use the mixed strategy

Since, we do not have saddle point Let p_1 the probability of Mr A selecting strategy I & Hence $(1 - p_1)$ be the probability of Mr A selecting strategy 2

If B select strategy	Expected pay off of A
1	$-5\ (p_1) + 8\ (1 - p_1) = -13p_1 + 8$
2	$5\ (p_1) - 4\ (1 - p_1) = 9p_1 - 4$
3	$0\ (p_1) + -1\ (1 - p_1) = p_1 - 1$
4	$-1\ (p_1) + 6\ (1 - p_1) = -7p_1 + 6$
5	$8\ (p_1) + -5\ (1 - p_1) = 13p_1 - 5$

We plot these values on the graph given below:



Since R is the maximin point and here B_1, B_3 intersect. These strategies will be selected & the resultant matrix is produced below

		B		
		1	3	Odds
A	1	-5	0	9
	2	8	-1	5
	odds	1	13	

$$V = \frac{a_1(b_1 - b_2) + b_1(a_1 - a_2)}{(b_1 - b_2) + (a_1 - a_2)} = \frac{(-5\times 9) + (8\times 5)}{9+5} = \frac{-5}{14}$$ **Ans.**

A		B	
Probability of selecting Strategies No.		**Probability of selecting Strategies No.**	
1	9/14	1	1/14
		2	0
2	5/14	3	13/14
		4	0
		5	0