CHAPTER-7: TRANSPORTATION PROBLEM

1. Introduction

The transportation problem is one of the subclass of LPPs in which the objective is to transport various quantities of a single homogeneous commodity (that are initially stored at various origins) to different destinations in such a way that the total transportation cost should be minimum.

Ex:

(1) A shoe company which has in diff. manufacturing units located at different places of India. The total products are to be transported to 'n' retail shops in 'n' different places of India.

(2) Similar case for cold drink company.

A transportation table is used to solve this transportation problem as shown below (like simplex table to solve LPP)

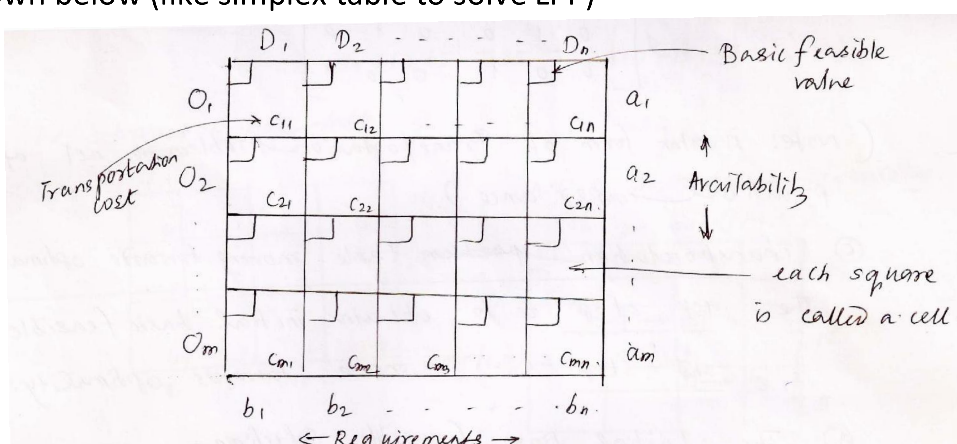

Fig 7.1: Transportation Table (m origin & n destination. Here m = 4, n = 5).

The various origins capacity and destination requirements are presented in fig. 7.1

A necessary and sufficient condition for the existence of a feasible solution to the transportation problem – $b_l + b_2 + b_3$ --- b_n = $a_l + a_2 + a_3$ --- am. i.e. total requirements = total availability.

Matrix form of transportation problem

Minimize z=cx $\quad c, x^T \in R^{mn}$

Subject to constraints

Ax = b $\quad x \geq o \; b^T \in R^{mn}$

Where x= [x_{ll} …… x_{ln}, x_{21} ….x_{2n} …. x_{ml}… x_{mn}]

b= [a_l, a_2--- a_m, b_l --- b_n], A is an (m+n) xmn real matrix containing the coefficient of the constraints and is the cost vector.

It should be noted that the elements of A are either O or +l. For a transportation problem involving 2 origin & 3 estimations (m=2, n=3) the matrix A is given by

$$A = \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ \hdashline 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

(Note: matrix form of transportation problem is not of practical importance).

Transportation table moving towards optimality The 1st step is to obtain initial basic feasible sol^n. With 2nd step " " move towards optimality.

The initial basic feasible solution

The various methods used to obtain the initial basic feasible are

(i) North west corner Rule
(ii) Row minima method
(iii) Column minima method
(iv) Matrix minima method
(v) Vogels approximation method

(i) North west Corner rule

In this method the first assignment is made is the cell occupying the upper left hard (north-west) corner of the transportation table. a sample problem will clarity this method/rule.

Determine the initial basic feasible solution to the following transportation problem.

	D1	D2	D3	D4	
O1	6	4	1	5	14
O2	8	9	2	7	16
O3	4	3	6	2	5
	6	10	15	4	

Requirements

Soln – Feasible solution exist if sum of availability = sum of requirements ie, 14+16+5 = 35 = 6+10+15+4

The transportation Table-

6	4	1	5	14
8	9	2	7	16
4	3	6	2	5
6	10	15	4	

Applying north-west corner rule →

6 →	8 ↓			14
	2 →	14 ↓		16
		1 →	4	5
6	10	15	4	

It has been proved theoretically that a feasible solution obtained by North-west corner rule (or by other method) is a basic solution.

(ii) Row minima method (with example)

Determine the initial basic feasible solution to the T.P. Using row minima method.

	A	B	C	Avai.
I	50	30	220	1
II	90	45	170	3
III	250	200	50	4
Req.	4	2	2	

Soln.

50	1 30	220
90	45	170
250	200	50

	1		
90	1 45	170	

→

	1		
2	1		2
250	200	50	
4		2	

	1		
2	1		
250	200	2 50	4
2		2	

→

	1		
2	1		
2		2	2
2			

Now Transportation cast = 1x30+2x80+1x45+2x250+2x50 = 855 unit cost/ Rs.

(iii) Column minima method

It is similar to row minima method.

(iv) Matrix minima method

Obtain an initial basic feasible solution using matrix minima method.

	D_1	D_2	D_3	D_4	
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
	4	6	8	6	24

Solution:

1	2	3	4	6
4	3	2	6 0	8
4 0	2	2	1	10
4	6	8	6	

→

	6 2	3		6
	3	2	6	2
4	6 2	2		6
	6	8		

	6						6			
		2	6	2	→			2	6
4	0	6		6		4	0	6	
		8							

The transportation cost (TL) = 2x6+2x2+6x0+4x0+0x2+6x2

= 12+4+12 = 28 (Ans)

(v) **Vogels Approximation method**

Ex.: Obtain initial basic feasible solution using vogel's approximation method.

5	1	3	3	34
3	3	5	4	15
6	4	4	3	12
4	-1	4	2	19
21	25	17	17	80

Solution:

5	1	3	3	34(2)
3	3	5	4	15(0)
6	4	4	3	12(1)
4	19 -1	4	2	19(3)
21 (2)	25 (2)	17 (1)	17 (0)	

→

5	6 1	3	3	34(2)
3	3	5	4	15(0)
6	4	4	3	12(1)
	19			
21 (2)	6 (2)	17 (1)	17 (0)	

5	6	3	3	28(0)
15 3		5	4	15(1)
6		4	3	12(1)
	19			
21 (2)		17 (1)	17 (0)	

← →

5	6	3	3	28(0)
15				
6		12 4	3	12(1)
	19			
6 (1)		17 (1)	17 (0)	

6 5	6 1	5 3	17 3	28
15 3	3	5	4	
6	4	12 4	3	
4	19 -1	4	2	
6		5	17	

Total cost = $6\times5+6\times1+5\times3+17\times3+15\times3+12\times4+19\times(-1)$

$=30+6+15+51+45+48-19 = 176.$

With the above techniques the initial basic feasible solution is obtained.

(B) Moving towards optimality

The procedure can be explained by taking the following example.

	D1	D2	D3	D4	Avai.
O1	6	4	1	5	14
O2	8	9	2	7	16
O3	4	3	6	2	5
Req.	6	10	15	4	

Step -1

The initial basic feasible solution is first obtained by any of the 5 methods. let by following the North-west corner rule the initial basic feasible sop –

6 6	8 4	 1	 5
 8	2 9	14 2	 7
 4	 3	1 6	4 2

Step – 2

The dual variables ui & vj are first calculated for all basic cells such that

ui+vj = aj for all basic cells. These are calculated by assuming u1=o as presented in the following.

$u_l+v_l= 6 \to o+v_l = 6 \to v_l = 6$

$u_l+v_2= 4 \to o+v_2 = 4 \to v_2 = 4$ and so or.

6 6	8 4	 1	 5	0
 8	2 9	14 2	 7	5
 4	 3	1 6	4 2	9
6	4	-3	-7	

Step – 3

Now compute the net evaluation zij for all non basic cells. i.e., zij-cij = vi+vj – cij for all nonbasic cells, and present in the non basic cell at the right top corner with paren these (or bracket), if all the values of net evaluation is –ve, then this is the optimal solution. Otherwise go to step -4. Following this rule the net evaluations for all and shown aside.

6 6	8 4	(-4) 1	(-12) 5	0
(3) 8	2 9	14 2	(-9) 7	5
(11) 4	(10) 3	1 6	4 2	9
6	4	-3	-7	

Step -4

Select the cell where highest positive number in the non basic cell exist. In the present problem. The cell (3,1) has the highest +ve number II. now a loop is constructed starting from this cell through the basic cells and ends in the starting cell. A small value Q will be added and substracted alternatively from each all as shown in the following.

6 - Ɵ 6	8 + Ɵ 4	(-4) 1	(-12) 5
(3) 8	2 - Ɵ 9	14 + Ɵ 2	(-9) 7
(11) + Ɵ 4	(10) 3	1 - Ɵ 6	4 2

The minimum value of Ɵ is min (6,8,2,14,1) = 1

(Note : A loop will be formed if it connect even no. of cells. here it is 6)

Step – 5

The new basic feasible solution and the net evaluation are presented in the following table.

5 6	9 4	(-4) 1	(-1) 5	0
(3) 8	1 9	15 2	(2) 7	5
1 4	(-1) 3	(-11) 6	4 2	-2
6	4	-3	4	

Step – 6

Select the cell where highest positive numbers in the non basic cell exist. In the present problem, the cell (3,1) has the highest +ve number II. Now a loop is constructed starting from this cell through the basic cells and ends in the starting cell. A small value Q will be added and substracted alternatively from each all as shown in the following.

5 – ϴ 6	9 + ϴ 4	(-4) 1	(-1) 5
(3) ϴ 8	1 - ϴ 9	15 2	(2) 7
4 4	3	6	4 2

→

4 6	10 4	1	5
1 8	9	15 2	7
1 4	3	6	4 2

Now the net evaluation for the table in prepare-

4 6	10 4	(-1) 1	(-1) 5	0
1 8	(-3) 9	15 2	(-1) 7	2
1 4	(-1) 3	(-8) 6	4 2	-2
6	4	0	4	

Since all the net evaluation are –ve, it is the optimal one. The optimum transportation cost = 4x6+10x4+1x8+15x2+1x4+4x2 = 114 Unit.

Degeneracy in transportation problem

If in a transportation problem (with on origin and 'n' destinations) the total numbers of positive basic variables < (m+n-I), then the transportation problem is a degenerate one.

It can develop in 2 ways :

(i) While determining the initial basic feasible solution following the 5 methods.

(ii) At some iteration stage used for getting the optimum solution.

How to remove/eliminate degeneracy?

A sample problem can be taken.

Prob.: obtain an optimum basic feasible solution to the following degenerate transportation problem.

7	3	4	2	
2	1	3	3	Available
3	4	6	5	
4	1	5	10	

Demand

Following the North-west corner rule an initial assignment in made as shown below.

2			2
2	1		3
		5	5
4	1	5	

In this case, the total no. of basic cells = 4, m+n-I = 5

Now 4 < 5

Hence the basic solution degenerate. in order to remove the degeneracy we require only one +ve variable. this is done in the following way.

Step – 1

A small +ve quantity € is selected and is allowed in the cell (2,3) as shown in the following table.

2 7	3	4	2
2 2	1 1	6 3	3+E
3	4	5 6	5
4	1	5+E	

It is seen that even after this, the basic cell do not form a loop i.e., the augmented solution remains basic (A feasible solution which does not form a loop is called basic)

Step – 2

The net evaluation is now computed and tested for optimality.

Since all the net evaluation for the non basic variables are not –ve, the initial solution is not optimum. Now the non basic cell (1,3) must enter the basic (because it has highest the value)

2 - Θ 7	(3) 3	Θ (4) 4	0
2 + Θ 2	1 1	Θ ≥0 3	-5
(2) 3	(0) 4	5 6	-2
uj 7	6	8	

The above integration is repeated.

2 - Θ 7	(3) 3	E + Θ 4	0
2 2	1 1	(-4) 3	-5
(6) Θ 3	(4) 4	5 - Θ 6	2
7	6	4	

Now Θ = 2, so the values in the table are:

(-6) 7	(-3) 3	2 4	0
2 - Θ 2	1 1	Θ (2) 3	1
2 + Θ 3	(-2) 4	3 – Θ	2
Uj 1	0	4	

(Taking Θ = 2)
→

	(-6) 7	(-1) 3	2 4	0
	(-2) 2	1 1	2 3	-1 ←optimal solution
	4 3	(0) 4	1 6	2
Uj	1	2	4	

& optimal transportation cost = 2×4+1×1+2×3+4×3+1×6 = 33 unit.