CHAPTER – 2: INVENTORY CONTROL

© 1. Introduction

Inventory is defined as the list of movable goods which are necessary to manufacture a product and to maintain the equipments and machinery in good working order/condition.

Classification

Broadly Classified into

- Direct inventory
- Indirect inventory

i. Direct inventory

It plays direct role in the manufacture of product such as:

- Raw materials
- Inprocess inventories (= work in progress)
- Purchased parts (purchasing of some components instead of manufacturing in the plant)
- Finished goods.

ii. Indirect inventory

It helps the raw materials to get converted into finished part. such as:

- Tools
- Supplies
  - miscellaneous consumable – brooms, cotton, wool, jute, etc.
  - welding electrode, solders etc.
  - abrasive mat – emery cloth, sand paper etc.
  - brushes, maps, etc.
  - oil greases etc.
  - general office supplies – candles, sealing wax etc.
  - printed forms such as – envelope, letter heads, quotation forms etc.
**Inventory control**

Inventory control means – making the desired items of required quality and quantity available to various departments/section as & when they need.

**(c) Relevant costs**

The relevant costs for how much & when decisions of normal inventory keeping one:

1. **Cost of capital**
   Since inventory is equivalent to locked-up working capital the cost of capital is an important relevant cost. this is the opportunity cost of investing in inventory.

2. **Space cost**
   Inventory keeping needs space and therefore, how much and when question of inventory keeping are related to space requirements. this cost may be the rent paid for the space.

3. **Materials handling cost**
   The material need to be moved within the warehouse and the factory and the cost associated with the internal movement of materials (or inventory) is called materials handling cost.

4. **Obsolescence, spoilage or Deterioration cost**
   If the inventory is procured in a large quantity, there is always a risk of the item becoming absolute due to a change in product design or the item getting spoiled because of natural ageing process. Such cost has a relation to basic question of how much and when?

5. **Insurance costs**
   There is always a risk of fire or theft of materials. a firm might have taken insurance against such mishaps and the insurance premium paid are the relevant cost.

6. **Cost of general administration**
   Inventory keeping will involve the use of various staffs. with large inventories, the cost of general administration might go up.

7. **Inventory procurement cost**
Cost associated with the procurement activities such as tendering, evaluation of bids, ordering, follow-up the purchase order, receipt and inspection of materials etc. is called inventory procurement cost.

(c) **Basic EOQ model**

EOQ = Economic Order Quantity.

EOQ represent the size of the order (or lot size) such that the sum of carrying cost (due to holding the inventory) and ordering cost is minimum. It is shown by point A of figure 2.1.

As mentioned earlier, the two most important decisions related to inventory control are:

- When to place an order? &
- How much to order?

In 1913, F.W. Harris developed a rule for determining optimum number of units of an item to purchase based on some fundamental
assumptions. This model is called Basic Economic Order Quantity model. It has broad applicability.

**Assumptions**

The following assumptions are considered for the sake of simplicity of model.

1) Demand (D) is assumed to be uniform.
2) The purchase price per unit (P) is independent of quantity ordered.
3) The ordering cost per order (Co) is fixed irrespective of size of lot.
4) The carrying cost/holding cost (Cc) is proportional to the quantity stored.
5) Shortage are not permitted i.e., as soon as the level of inventory reaches zero, the inventory is replenished.
6) The lead time (LT) for deliveries (i.e. the time of ordering till the material is delivered) is constant and is known with certainty.

The assumptions 5 and 6 are shown graphically in fig 2.2.

Let Q = order size

Therefore, the number orders/year = \( \frac{D}{Q} \) --------(1)
Average inventory level = \( \frac{Q}{2} \)

Ordering cost per year = \( \frac{D}{Q} \times C_o \) \( \text{-------}(2) \)

Carrying cost per year = \( \frac{Q}{2} \times C_c \) \( \text{-------}(3) \)

Purchase cost/year = \( D \times P \) \( \text{-------}(4) \)

Now, the total inventory cost per year = \( TC = \frac{D}{Q} \times C_o + \frac{Q}{2} \times C_c + D \times P \) \( \text{-------}(5) \)

Differentiating Eq (5) w.r.t. Q it becomes:

\[
\frac{d(TC)}{dQ} = -\frac{D}{Q^2} \times C_o + \frac{Q}{2} \times C_c \quad \text{-------}(6)
\]

The 2nd derivative = \( +\frac{2D}{Q^3} \times C_o \) \( \text{-------}(7) \)

Since the 2nd derivative is +ve, we can equate the value of first derivative to zero to get the optimum value of Q.

i.e., \( -\frac{D}{Q^2} \times C_o + \frac{Q}{2} \times C_c = 0 \)

\[\therefore \frac{Q}{2} \times C_c = \frac{D}{Q^2} \times C_o\]

\[\therefore Q^2 = \frac{2CoD}{Cc}\]

\[\therefore Q = \sqrt{\frac{2CoD}{Cc}} \quad \text{-------}(8)\]

So, optimum \( O = EOQ = \sqrt{\frac{2CoD}{Cc}} \)

Ex: ABC company estimates that it will sell 12000 units of its product for the forthcoming year. The ordering cost is Rs 100/- per order and the carrying cost per year is 20% of the purchase price per unit. The purchase price per unit is Rs 50/-.

Find
i. Economic Order Quantity
ii. No. of orders/year
iii. Time between successive order.

Solution:

Given $D = 12000$ units/yr, $C_o = Rs 100$/year
$Cc = Rs 50 \times 0.2 = Rs 10/-$ per unit/year

Therefore (i) $EOQ = \sqrt{\frac{2CD}{Cc}} = \sqrt{\frac{2 \times 100 \times 12000}{10}} = 490$ units approx.

No. of orders/year $= \frac{D}{Q*} = \frac{12000}{490} = 24.49$

Time between successive order $= \frac{Q*}{D} = \frac{490}{12000} = 0.04$ year $= 0.48$ month

© Models with Quantity Discount

When items are purchased in bulk, buyers are usually given discount in the purchase price of goods. this discount may be a step function of purchase quantity as stated in the Following.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Purchase price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq Q_1 &lt; b_1 \rightarrow$</td>
<td>$P_1$</td>
</tr>
<tr>
<td>$b_1 \leq Q_2 &lt; b_2 \rightarrow$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>$b_{n-1} \leq Q_n \rightarrow$</td>
<td>$P_n$</td>
</tr>
</tbody>
</table>

The procedure to compute the optimal order size for this situation is given in the following steps.

Step- 1
Find EOQ for nth (last) price break

\[ Q^*_n = \sqrt{\frac{2CoD}{i^*Pn}} \]

Where \( i = \) fraction of purchase price for inventory carrying.

If it is greater than or equal so \( b_{n-1} \), then the optimal order size \( Q = Q_n \); otherwise go to step-2.

**Step-2**

Find EOQ for (n-1)th price break

\[ Q^*_{n-1} = \sqrt{\frac{2CoD}{iP_{n-1}}} \]

If it is greater than or equal to \( b_{n-2} \), then compute the following and select the least cost purchase quantity as optimal order size; otherwise go to step-3

i) \( TC(Q_{n-1}) \)
ii) \( TC(b_{n-1}) \)

**Step-3**

Find EOQ for (n-2)th price break

\[ Q^*_{n-2} = \sqrt{\frac{2CoD}{iP_{n-2}}} \]

If it is greater than or equal to \( b_{n-3} \), then compute the following and select the least cost purchase quantity; otherwise go to step-4.

i) Total cost, \( TC(Q^*_{n-2}) \)
ii) Total cost, \( TC(b_{n-2}) \)
iii) Total cost, \( TC(b_{n-1}) \)

**Step-4**

Continue in this manner until \( Q^*_{n-k} \leq b_{n-k-1} \). Then compare total cost \( \pi(Q^*_{n-k}), \pi(b_{n-k}), \pi(b_{n-k+1}) \ldots \ldots. \pi(b_{n-1}) \) corresponding to purchase quantities
Q_{n-k}, b_{n-k}, b_{n-k+1}, \ldots, b_{n-1} \text{ respectively. Finally select the purchase quantity w.r.t. minimum total cost.}

Ex: Annual demand for an item is 4800 units. ordering cost is Rs 500/- per order. inventory carrying cost is 24% of the purchase price per unit. the price break are given below.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; Q_1 &lt; 1200(b_1)</td>
<td>10</td>
</tr>
<tr>
<td>1200 ≤ Q_2 &lt; 2000(b_2)</td>
<td>9</td>
</tr>
<tr>
<td>2000 ≤ Q_3 (b_3)</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) Find optimal order size.

Solution (a) D = 4800, Co = 500, I = 0.24

**Step- 1**  
P_3 = Rs 8/-  
Q_3 = \sqrt{\frac{2 \cdot Co \cdot D}{IP_3}} = \sqrt{\frac{2 \times 500 \times 4800}{0.24 \times 8}} = 1581

Since Q_3 < b_2, i.e., 2000, go to step-2

**Step- 2**  
P_2 = Rs 9/-  
Q_2 = \sqrt{\frac{2 \cdot Co \cdot D}{IP_2}} = \sqrt{\frac{2 \times 500 \times 4800}{0.24 \times 9}} = 1491

Since Q_2 > b_1 i.e., 1200, find the following costs & select the order size based on least cost.

\[ TC(Q_2) = 9 \times 4800 + 500 \times \frac{480000}{1491} + \frac{0.24 \times 9 \times 1491}{2} = Rs \ 46420 \ (approx) \]
\[ TC(b_2) = 8 \times 4800 + 500 \times \frac{480000}{2000} + \frac{0.24 \times 8 \times 2000}{2} = Rs \ 41,520 \ (approx) \]

The least cost is Rs 41,520, Hence optimal order size is 2000.

© **Economic Batch Quantity**

(a) Without shortage.

It is a manufacturing model
(b) With shortage.

(a) Manufacturing model without shortage

if a company manufacture its component which is required for its main product, then the corresponding model of inventory is called → manufacturing model. This model will be without/with shortage. The rate of consumption of item is uniform throughout the year. The cost of production per unit is same irrespective of production 10+ size. Let,

\[ r = \text{annual demand of an item} \]
\[ k = \text{production rate of item (No. of units produced per year)} \]
\[ C_0 = \text{cost per set-up.} \]
\[ C_c = \text{carrying cost per unit per period.} \]
\[ P = \text{cost of production per unit} \]
\[ \text{EOQ} = \text{Economic Batch Quantity} \]

The variation of inventory with time without shortage is shown below.

During the period \( t_1 \), the item is produced at the rate of \( k \) units per period and simultaneously it is consumed at the rate of \( r \) units per period. So during this period, the inventory is built at the rate of \( (k-r) \) units per period.

During the period \( t_2 \), the production of items is discontinued but the consumption of item is continued. Hence the inventory is decreased at the of \( r \) units per period during this period.
The various formulae for this situation

→ Economic Batch Quantity (EBQ) = \( \sqrt{\frac{2 Cor}{Cc (1-r/k)}} \)

\[ t1^* = \frac{Q^*}{K} \]

\[ t2^* = \frac{Q*[1-r/k]}{r} \]

Cycle time = \( t1^* + t2^* \) (Refer Operation Research Book by Kanti Swarup for detail)

Ex: If a product is to be manufactured within the company, the details are as follows:

- \( r = 24000 \) units/year
- \( k = 48000 \) units/year
- \( Co = Rs 200/- \) per set-up
- \( Cc = Rs 20/- \) per unit/year

**Find the EBQ & Cycle time**

Solution:

\[ EBQ = \sqrt{\frac{2 \times 200 \times 24000}{20(1-24000/480000)}} = 980 \text{ approx.} \]

\[ t1^* = \frac{Q^*}{k} = \frac{98000}{48000} = 0.24 \text{ yr} = 0.24 \text{ month} \]

\[ t2^* = \frac{Q^*}{r} \left( 1 - \frac{r}{k} \right) = \frac{98000}{24000} \left( 1 - \frac{24000}{48000} \right) = 0.24 \text{ yr} = 0.24 \text{ month} \]

Total cycle time = \( t1^* + t2^* = 0.24 + 0.24 = 0.48 \text{ month} \)

**(b) Manufacturing model with shortage**

In this model, the items are produced and consumed simultaneously for a portion of cycle time. The rate of consumption of items is uniform throughout the year. The cost of production per unit is the same irrespective of production lot size. In this model, stock out/shortage is permitted. It is assumed that the stock out units will be satisfied from the units which will be produced at a later
date with a penalty (like rate reduction). This is called back ordering. The operation of this model is shown in fig. 2.4.

The variables which are used in this model are given below.

- \( r \) = annual demand of an item
- \( k \) = production rate of the item
- \( C_o \) = cost/setup
- \( C_c \) = carrying cost/unit/period
- \( C_s \) = shortage cost/unit/period.
- \( P \) = cost of production per unit

In the above model

- \( Q \) = Economic batch quantity
- \( Q_1 \) = Maximum inventory
- \( Q_2 \) = Maximum stock out

By applying mathematics

\[ Q^* = EBQ = \sqrt{\frac{2 C_o k r}{C_c (k-r)}} \frac{(C_c+C_s)}{C_s} \]  \( \text{(1)} \)
\[ Q^*1 = \sqrt{\frac{2 \cdot Cc \cdot r(k-r)}{Cc \cdot k}} \times \frac{Cs}{Cc+Cs} \]  \hspace{1cm} (2)

\[ Q^*2 = \sqrt{\frac{2 \cdot Co \cdot Cc}{Cs(Cc+Cs)}} \times \frac{r(k-r)}{k} \]

Also \( Q^*1 = \left( \frac{k-r}{k} \cdot Q^* \right) - Q^*_2 \)

\[ t^* = \frac{Q^*}{r}; t^*_1 = \frac{Q^*_1}{k-r}; t^*_2 = \frac{Q^*_1}{r}; t^*_3 = \frac{Q^*_2}{r}; t^*_4 = \frac{Q^*_2}{(k-r)} \]

(c) **Periodic and Continuous Review system for stochastic system (= probabilities)**

The situation where demand is not known exactly but the probability distribution of demand is known (from previous data) is called a stochastic system/problem.

The control variable in such case is assumed to be either

- The scheduling period. or
- The order level. or
- Both.

The optimum order level will thus be derived by minimizing the total expected cost rather than the actual cost involved.

**Stochastic problem with uniform demand**

The following assumptions are made for the simplicity of model.

1) Demand is uniform over a period (let \( r \) unit/period)
2) Reorder time is fixed and known
3) Production of commodities is instaneous, and
4) Lead time is negligible.

Let

- (i) the holding cost/carrying cost per unit item = \( Cc \)
- (ii) the shortage cost/item/time = \( Cs \)
- (iii) the inventory level at any time \( t = Q \)
The problem is to determine the optimum order level \( Q \) (without shortage) at the beginning of each period, where \( Q \geq r \) or \( Q < r \) (with shortage). In both these cases the inventory system is shown in fig. 2.5 below.

The units that build up an inventory may consist of either discrete (= periodic) / continuous system.

(A) **Periodic system (or Discrete unit)**

Let the demand for \( r \) unit be estimated at a discontinuous rate with probability \( P(r) \); \( r = 1, 2, 3, \ldots \). That is we may expect demand for 1 unit with probability, \( p(1) \); 2 units with probability \( p(2) \); and so on. Since all the possibilities are taken care of, we must have

\[
\sum_{r=1}^{\infty} P(r) = 1 \text{ and } P(r) \geq 0
\]

Penalty costs are associated with producing \( Q \) which is less than the amount actually demanded (i.e. \( Q < r \)). It is denoted by the shortage cost \( (Cs) \). This may be made up of either

i. Loss of good will &  
ii. Contract penalty for failure to deliver.

Similarly we assume that the penalty costs are associated with producing \( Q \), which is lying surplus even after meeting the demand (i.e. \( Q \geq r \)). We denote this cost by \( Cc \), as unit cost of oversupplying. This may be made up of either

i) Loss, when extra items are to be sold at lesser price. &  
ii) Held by the producer incurring cost.
Clearly these costs entirely depend upon the discrepancy between Q & demand (r). And then discrepancy is an under/oversupply.

Expected size of over supply = \( \sum_{r=1}^{Q} (Q - r) P(r) \) \( \text{----(1)} \)

Expected cost of over production = \( Cc \sum_{r=1}^{Q} P(r) \) \( \text{----(2)} \)

Expected size of undersupply = \( \sum_{r=Q+1}^{\infty} (r - Q) P(r) \) \( \text{----(3)} \)

Expected cost of undersupply = \( Cs \sum_{r=Q+1}^{\infty} (r - Q) P(r) \) \( \text{----(4)} \)

Thus the total expected cost

\( TEC(Q) = Cc \sum_{r=1}^{Q} (Q - r) P(r) + Cs \sum_{r=Q+1}^{\infty} (r - Q) P(r) \) \( \text{----(5)} \)

The problem now is to find Q so as to minimize TEC(Q).

Let on amount Q+1 instead of Q be produced. Then the total expected cost equation \( \rightarrow \)

\( TEC(Q) = Cc \sum_{r=1}^{Q+1} (Q + 1 - r) P(r) + Cs \sum_{r=Q+2}^{\infty} (r - Q - 1) P(r) \) \( \text{----(6)} \)

On simplification (referring O.R. by Kanti Swarup)

\( TEC(Q+1) = TEC(Q) + (Cc + Cs) \sum_{r=1}^{Q} P(r) - Cs \)

& \( TEC(Q+1) = TEC(Q) + (Cc + Cs) P(r \leq Q) - Cs \) \( \text{----(7)} \)

Similarly, when an amount Q-1, instead of Q is produced,

\( TEC(Q-1) = TEC(Q) - (Cc + Cs) P(r \leq Q-1) + Cs \) \( \text{----(8)} \)

Suppose that we find Q* having the property that

(i) \( TEC(Q^*) < TEC(Q^*+1) \) &

(ii) \( TEC Q^* < TEC (Q^*-1) \),

Then Q* would clearly represent a local minimum for TEC(Q)

Let us define \( \Delta TEC(Q) = TEC(Q+1) - TEC(Q) \) as the difference between the total expected cost for Q and for the next higher value (Q+1). Thus from Eq(7) & (8), we have

\( \Delta [TEC(Q)] = (Cc + Cs) P(r \leq Q) - Cs \)
And $\Delta[TEC(Q-1)] = (C_c + C_s) P(r \leq Q-1) - C_s$

Therefore, if $Q^*$ be the local minima for $TEC(Q)$, then

(i) $TEC(Q^*) < TEC(Q^*+1) \quad :- \Delta[TEC(Q^*)] > 0$

\[ :- (C_c + C_s) P(r \leq Q) - C_s > 0 \]

\[ :- P(r \leq Q) > \frac{C_s}{C_c + C_s} \quad (9) \]

And (ii) $TEC(Q^*) < TEC(Q^*-1) \quad :- \Delta[TEC(Q^*-1)] < 0$

\[ :- (C_c + C_s) P(r \leq Q-1) - C_s < 0 \]

\[ :- P(r \leq Q-1) < \frac{C_s}{C_c + C_s} \quad (10) \]

From Eq (9) & (10)

\[ P(r \leq Q-1) < \frac{C_s}{C_c + C_s} < P(r \leq Q) \]

Hence if the oversupply cost $C_c$ and the shortage cost $C_s$ are known, the optimal quantity $Q^*$ is determined when the value of cumulative probability distribution $P(r)$ just exceeds the ratio $\frac{C_s}{C_c + C_s}$. That is $Q^*$ is determined by comparing a cost ratio with probability figure.

(B) continuous Review System

When certain demand estimated as a continuous random variable, the cost equation of the inventory involves integrals instead of summation sign. The discrete point probabilities $p(r)$ are replaced by probability differential $f(r)$ for small interval, say $r \pm \frac{dr}{2}$ of continuous demand variable. In this case

\[ \int_{1}^{\infty} f(r)dr = 1 \quad \text{and} \quad f(r) \geq 0. \]

Proceeding exactly in the manner as (A), Let

\[ Q = \text{quantity produced} \]

\[ C_c = \text{penalty cost per unit cost of oversupply (} Q \geq r), \text{ and} \]

\[ C_s = \text{penalty cost per unit cost of under supply (} Q < r) \]

The expected sizes of over and under supply are:
$$\int_{r=1}^{Q} (Q - r) F(r) \, dr \text{ and } \int_{r=Q}^{\infty} (r - Q) f(r) \, dr \text{ respectively}$$

The total expected cost (TEC) associated with producing an amount $Q$ when facing a demand known only as a continuous random variable is given by:

$$\text{TEC}(Q) = C_c \int_{r=1}^{Q} (Q - r) F(r) \, dr + C_s \int_{Q}^{\infty} (r - Q) f(r) \, dr \quad \text{------(11)}$$

We now determine optimum value $Q^*$ so as to minimize $\text{TEC}(Q)$

$$\frac{\partial \text{TEC}(Q)}{\partial Q} = \frac{\partial }{\partial Q} \{ C_c \int_{1}^{Q} (Q - r) f(r) \, dr + C_s \int_{Q}^{\infty} (r - Q) f(r) \, dr \}$$

$$= C_c \left[ \int_{1}^{Q} f(r) \, dr \right] + (Q - Q) \cdot 1 - (Q - 1) f(1) \cdot 0 + C_s \left[ - \int_{Q}^{\infty} f(r) \, dr + \int_{Q}^{\infty} (r - Q) f(r) \, dr \right]$$

$$= C_c \int_{1}^{Q} f(r) \, dr - C_s \int_{Q}^{\infty} f(r) \, dr$$

$$= C_c \int_{1}^{Q} f(r) \, dr - C_s \int_{1}^{Q} f(r) \, dr$$

$$= (C_c + C_s) \int_{1}^{Q} f(r) \, dr - C_s \quad \text{since} \int_{1}^{\infty} f(r) \, dr = 1$$

$$\Rightarrow \frac{\partial \text{TEC}(Q)}{\partial Q} = 0 = (C_c + C_s) \int_{1}^{Q} f(r) \, dr = C_s \int_{1}^{Q} f(r) \, dr = \frac{C_s}{C_c + C_s}$$

& $P(r \leq Q) = \frac{C_s}{C_c + C_s} \quad \text{----------(12)}$

Moreover,

$$\frac{\partial^2 \text{TEC}(Q)}{\partial Q^2} = (C_c + C_s) f(Q) > 0$$

Thus $Q$ as determined by Eq (12) is an optimal value so as to minimize $\text{TEC}(Q)$.

Hence $P(r \leq Q) = F(Q^*) = \frac{C_s}{C_c + C_s} \quad \text{-------(13)}$

Where $F(Q) = \int_{1}^{Q} f(r) \, dr$

This indicates that "the best quantity to be produced is that value of $Q$ for which the value of cumulative probability distribution of $r$ is equal to $\frac{C_s}{C_c + C_s}$".
The optimum value of Q for continuous demand variable may be illustrated graphically as shown below.

Ex: A newspaper boy buys paper for Rs 1.40 and sells them for Rs 2.45. He can't return unsold newspaper. Daily demand has the following distributions.

<table>
<thead>
<tr>
<th>Number of customers</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.03</td>
<td>0.05</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.15</td>
<td>0.12</td>
<td>0.10</td>
<td>0.10</td>
<td>0.07</td>
<td>0.06</td>
<td>0.02</td>
</tr>
</tbody>
</table>

If each days demand is independent of the previous days, how many papers he should order each days?

**Solution:**

Given Cc = Rs1.40, Cs = Rs2.45 – 1.40 = Rs1.05

The point probabilities are:

<table>
<thead>
<tr>
<th>r</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(r)</td>
<td>0.03</td>
<td>0.05</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.15</td>
<td>0.12</td>
<td>0.10</td>
<td>0.10</td>
<td>0.07</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>Σ P(r)</td>
<td>0.03</td>
<td>0.08</td>
<td>0.13</td>
<td>0.23</td>
<td>0.38</td>
<td>0.53</td>
<td>0.65</td>
<td>0.75</td>
<td>0.85</td>
<td>0.92</td>
<td>0.98</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Now \( \frac{Cs}{Cc+Cs} = \frac{1.05}{1.40+1.05} = 0.4285 \)

Now 0.38 < 0.4285 < 0.53 ; so the number of newspaper ordered = 30.0

(C) **Safety stock, Recorder point and Order Quantity Calculation**
The safety stock may be defined as minimum additional inventory to serve as a safety margin (or cushion) to meet unanticipated increase in usage resulting from various uncontrollable factors like

i) An unusual high demand
ii) Late receipt of incoming inventory

The reorder level (ROL) = DLT + safety stock (SS)

Where DLT = Demand during lead time

= Demand rate × Lead time period (from geometry)

= \( \frac{Demand}{day} \times LT \) in days.

Safety Stock (SS) = \( k \sigma \) where \( k \) = standard normal statistic value for a given service level & \( \sigma \) = standard deviation.

Ex: A firm has a demand distribution during a constant lead time with a standard deviation of 250 units. The firm wants to provide 98% service

a) How much safety stock should be carried.

b) If the demand during lead time averages 1200 units, what is the appropriate reorder level (ROL)?

Corresponding to 98% service level, K value from normal distribution table = 2.05.

Solution:

a) Safety stock (SS) = \( k \sigma = 2.05 \times 250 = 512 \) units

b) ROL = DLT + SS = 1200 + 512 = 1712 units

(C) ABC Analysis

ABC means → Always Better Control

ABC analysis divides inventories into three groupings in terms of percentage of number of items and percentage of total value. In ABC analysis important
items (high usage valued items) are grouped in C and the remaining middle level items are considered 'B' items.

The inventory control is exercised on the principle of "management by exception" i.e., rigorous controls are exercised on A items and routine loose controls for C items and moderate control in 'B' items. The items classified by virtue of their uses as:

<table>
<thead>
<tr>
<th>Category</th>
<th>% of items (approx)</th>
<th>% value (approx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A – High value items</td>
<td>10</td>
<td>70</td>
</tr>
<tr>
<td>B – Medium value items</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>C – Low value items</td>
<td>70</td>
<td>10</td>
</tr>
</tbody>
</table>

**Control policies for A items**

i) 'A' items are high valued items hence should be ordered in small quantities in order to reduce capital blockage.

ii) The future requirement must be planned in advanced so that required quantities arrive a little before they are required for consumptions.

iii) Purchase and stock control of A items should be taken care by top executives in purchasing department.

iv) Maximum effort should be made to expedite the delivery.

v) The safety stock should be as less as possible (15 days or less).

vi) 'A' items are subjected to tight control w.r.t.
   - Issue
   - Balance
   - Storing method

vii) Ordering quantities, reorder point and maximum stock level should be revised more frequently.

**Control policies for 'C' items**

i) The policies for 'B' items are in general between A & C.

ii) Order for these items must be placed less frequently.

iii) Safety stock should be medium (3 months or less).

iv) 'B' items are subjected to moderate control.
Control policies for 'C' items

i) 'C' items are low valued items.
ii) Safety stock should be liberal (3 months or more).
iii) Annual or 6 monthly order should be placed to reduce paper work & ordering cost and to get the advantage of discount.
iv) In case of these items only routine check is required.

Steps for ABC Analysis

1. Calculate the annual usage in units for each items.
2. Calculate the annual usage of each item in terms of rupees.
3. Rank the items from highest annual usage in rupees to lowest annual usage in rupees.
4. Compute total rupees.
5. Find the % of high, medium and low valued items in terms of total value of items.

The following example will give a clear and wide information about ABC analysis. Prepare ABC analysis on the following sample of items in an inventory.

<table>
<thead>
<tr>
<th>Item</th>
<th>Annual usage unit</th>
<th>Unit cost (Rs)</th>
<th>Annual usage (Rs)</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>30,000</td>
<td>0.01</td>
<td>300</td>
<td>6</td>
</tr>
<tr>
<td>b</td>
<td>2800</td>
<td>1.5</td>
<td>4200</td>
<td>1</td>
</tr>
<tr>
<td>c</td>
<td>300</td>
<td>0.10</td>
<td>30</td>
<td>9</td>
</tr>
<tr>
<td>d</td>
<td>1100</td>
<td>0.5</td>
<td>550</td>
<td>4</td>
</tr>
<tr>
<td>e</td>
<td>400</td>
<td>0.05</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>f</td>
<td>2200</td>
<td>1.0</td>
<td>2200</td>
<td>2</td>
</tr>
<tr>
<td>g</td>
<td>1500</td>
<td>0.05</td>
<td>75</td>
<td>8</td>
</tr>
<tr>
<td>h</td>
<td>8000</td>
<td>0.05</td>
<td>400</td>
<td>5</td>
</tr>
<tr>
<td>i</td>
<td>3000</td>
<td>0.30</td>
<td>900</td>
<td>3</td>
</tr>
<tr>
<td>j</td>
<td>800</td>
<td>0.10</td>
<td>80</td>
<td>7</td>
</tr>
</tbody>
</table>

Table showing ABC Analysis (ABC Ranking)

<table>
<thead>
<tr>
<th>Item</th>
<th>Annual usage (Rs)</th>
<th>Cumulative amount</th>
<th>Cumulative %</th>
<th>Ranking</th>
<th>Annual usage units</th>
<th>Cumulative %</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>4200</td>
<td>4200</td>
<td>47.97</td>
<td>A</td>
<td>2800</td>
<td>5.88</td>
</tr>
</tbody>
</table>
Accordingly a graph can be plotted.

**Benefits of ABC Analysis (By a suitable example)**

A company that has not made ABC analysis of its inventory makes 4 orders/year in respect of each item to get 3 months supply of every item. Taking a sample of 3 items, with different levels of annual consumptions, their average inventory (which is one half of order quantity) is worked out in the following table.

<table>
<thead>
<tr>
<th>Item</th>
<th>Annual consumption</th>
<th>No. of orders</th>
<th>Average working inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40,000</td>
<td>4</td>
<td>( \frac{10,000}{2} = 5000 )</td>
</tr>
<tr>
<td>B</td>
<td>4000</td>
<td>4</td>
<td>( \frac{1000}{2} = 500 )</td>
</tr>
<tr>
<td>C</td>
<td>400</td>
<td>4</td>
<td>( \frac{100}{2} = 50 )</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>12</strong></td>
<td><strong>5550</strong></td>
</tr>
</tbody>
</table>

But keeping the same no. of orders/year (i.e. 12), inventory can be reduced by 39% by segregating them according to their usage value (ABC analysis) as illustrated in the following table.

<table>
<thead>
<tr>
<th>Item</th>
<th>Annual consumption</th>
<th>No. of orders</th>
<th>Average working inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40,000</td>
<td>8</td>
<td>( \frac{5000}{2} = 2500 )</td>
</tr>
<tr>
<td>B</td>
<td>4000</td>
<td>3</td>
<td>( \frac{1333}{2} = 667 )</td>
</tr>
<tr>
<td>C</td>
<td>400</td>
<td>1</td>
<td>( \frac{400}{2} = 200 )</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>12</strong></td>
<td><strong>3367</strong></td>
</tr>
</tbody>
</table>
Thus the investment on inventory is reduced.

**Applications of ABC analysis**

ABC analysis can be effectively used in materials management. Such as

- Controlling raw materials components.
- Controlling work in progress inventories.

**Limitations of ABC analysis**

1. ABC analysis does not consider all relevant problems of inventory control such as a firm handling adequately low valued 'C' items.
2. ABC analysis is not periodically revised for which 'C' items like diesel oil in a firm will become most high valued items during power crisis.
3. The importance of an item is computed based on its consumption value and not its criticality.