FLUIDS MECHANICS

Unit-I: PROPERTIES OF FLUIDS

Fundamental Concepts:

- Mechanics : Deals with action of forces on bodies at rest or in motion.
- **State of rest and Motion:** They are relative and depend on the frame of reference. If the position with reference to frame of reference is fixed with time, then the body is said to be in a state of rest. Otherwise, it is said to be in a state of motion.
- Scalar and heater quantities: Quantities which require only magnitude to represent them are called scalar quantities. Quantities which acquire magnitudes and direction to represent them are called vector quantities.

Eg: Mass, time internal, Distance traveled \rightarrow Scalars

Weight, Displacement, Velocity \rightarrow Vectors

• Displacement and Distance



Unit: m

• Velocity and Speed: Rate of displacement is called velocity and Rate and distance traveled is called Speed.

Unit: m/s



- Acceleration: Rate of change of velocity is called acceleration. Negative acceleration is called retardation.
- Momentum: The capacity of a body to impart motion to other bodies is called momentum.

The momentum of a moving body is measured by the product of mass and velocity the moving body

Momentum = Mass x Velocity

Unit: Kgm/s

- Newton's first law of motion: Every body continues to be in its state of rest or uniform motion unless compelled by an external agency.
- Inertia: It is the inherent property the body to retain its state of rest or uniform motion.
- Force: It is an external agency which overcomes or tends to overcome the inertia of a body.
- Newton's second law of motion: The rate of change of momentum of a body is directly proportional to the magnitudes of the applied force and takes place in the direction of the applied force.



• Measurement of force:



Change in momentum in time 't' = mv - muRate of change of momentum = $\frac{mv - mu}{t}$

$$F \alpha \frac{mv - mu}{t}$$
$$F \alpha m \left(\frac{v - u}{t}\right)$$
$$F \alpha ma$$
$$F = K ma$$

If F = 1 When m = 1 and u = 1then K = 1 \therefore F = ma.

Unit: newton (N)

- Mass: Measure of amount of matter contained by the body it is a scalar quantity. Unit: Kg.
- Weight: Gravitational force on the body. It is a vector quantity.

F = ma W = mgUnit: newton (N) $g = 9.81 \text{ m/s}^2$

• Volume: Measure of space occupied by the body.

Unit: m^3 1 $m^3 = 1000$ litres



Work: Work done = Force x Displacement → Linear motion.
 Work done = Torque x Angular displacement → Rotatory motion.

Unit: Nm or J

• Energy: Capacity of doing work is called energy.

Unit: Nm or J

Potential energy = mgh



Kinetic energy =	$\frac{1}{2}$ mv ² or $\frac{1}{2}$	mr ω^2
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Power: Rate of doing work is called Power.

Power: =
$$\frac{\text{Force x displacement}}{\text{time}}$$

= Force x Velocity \rightarrow Linear Motion.
P = $\frac{2\Pi NT}{60} \rightarrow$ Rotatory Motion.

- Matter: Anything which possess mass and requires space to occupy is called matter.
- States of matter:

Matter can exist in the following states

- Solid state.
- ♦ Fluid state.



- Solid state: In case of solids intermolecular force is very large and hence molecules are not free to move. Solids exhibit definite shape and volume. Solids undergo certain amount of deformation and then attain state of equilibrium when subjected to tensile, compressive and shear forces.
- Fluid State: Liquids and gases together are called fluids. Incase of liquids Intermolecular force is comparatively small. Therefore liquids exhibit definite volume. But they assume the shape of the container

Liquids offer very little resistance against tensile force. Liquids offer maximum resistance against compressive forces. Therefore, liquids are also called incompressible fluids. Liquids undergo continuous or prolonged angular deformation or shear strain when subjected to tangential force or shear force. This property of the liquid is called flow of liquid. Any substance which exhibits the property of flow is called fluid. Therefore liquids are considered as fluids.

In case of gases intermolecular force is very small. Therefore the molecules are free to move along any direction. Therefore gases will occupy or assume the shape as well as the volume of the container.

Gases offer little resistance against compressive forces. Therefore gases are called compressible fluids. When subjected to shear force gases undergo continuous or prolonged angular deformation or shear strain. This property of gas is called flow of gases. Any substance which exhibits the property of flow is called fluid. Therefore gases are also considered as fluids.



• Branches of Mechanics:



- Fluid Statics deals with action of forces on fluids at rest or in equilibrium.
- Fluid Kinematics deals with geometry of motion of fluids without considering the cause of motion.
- Fluid dynamics deals with the motion of fluids considering the cause of motion.



Properties of fluids:

1. Mass density or Specific mass (ρ):

Mass density or specific mass is the mass per unit volume of the fluid.

$$\therefore \quad \rho = \frac{Mass}{Volume}$$
$$\rho = \frac{M}{V} \text{ or } \frac{dM}{dV}$$

Unit: kg/m³ or kgm³

With the increase in temperature volume of fluid increases and hence mass density decreases.

In case of fluids as the pressure increases volume decreases and hence mass density increases.

2. Weight density or Specific weight (γ):

Weight density or Specific weight of a fluid is the weight per unit volume.

$$\therefore \quad \gamma = \frac{Weight}{Volume}$$
$$\gamma = \frac{W}{V} \text{ or } \frac{dW}{dV}$$

Unit: N/m^3 or Nm^{-3} .

With increase in temperature volume increases and hence specific weight decreases.

With increases in pressure volume decreases and hence specific weight increases.

Note: Relationship between mass density and weight density:



We have
$$\gamma = \frac{Weight}{Volume}$$

 $\gamma = \frac{mass \ x \ g}{Volume}$
 $\gamma = \rho \ x \ g$

3. Specific gravity or Relative density (S):

It is the ratio of specific weight of the fluid to the specific weight of a standard fluid.

 $S = \frac{\gamma \text{ of fluid}}{\gamma \text{ of stan dard fluid}}$

Unit: It is a dimensionless quantity and has no unit.

In case of liquids water at 4°C is considered as standard liquid.

<u> γ (specific weight)</u> of water at 4°C (standard liquid) is 9.81 $\frac{kN}{m^3}$ or 9.81 x 10³ $\frac{kN}{m^3}$

Note: We have

1.
$$S = \frac{\gamma}{\gamma_{s \tan dard}}$$

 $\therefore \gamma = S \times \gamma_{s \tan dard}$

2.
$$S = \frac{\gamma}{\gamma_{s \tan dard}}$$

$$S = \frac{\rho x g}{\rho_{s \tan dard} x g}$$

$$S = \frac{\rho}{\rho_{s \tan dard}}$$



 \therefore Specific gravity or relative density of a fluid can also be defined as the ratio of mass density of the fluid to mass density of the standard fluid. Mass density of standard water is 1000 kg/m³.

$$\rho = S x \gamma_{s \tan dard}$$

4. Specific volume (\forall): It is the volume per unit mass of the fluid.

$$\therefore \quad \forall = \frac{Volume}{mass}$$
$$\forall = \frac{V}{M} \text{ or } \frac{dV}{dM}$$

Unit: m³/kg

As the temperature increases volume increases and hence specific volume increases. As the pressure increases volume decreases and hence specific volume decreases.



Problems:

1. Calculate specific weight, mass density, specific volume and specific gravity of a liquid having a volume of 4m³ and weighing 29.43 kN. Assume missing data suitably.

To find ρ - Method 1:

W=mg

- $29.43 \times 10^3 = mx9.81$ Method 2:
- m=3000kg

$$\therefore \rho = \frac{m}{v} = \frac{3000}{4}$$
 $\rho = 750 \text{ kg/m}^3$

 $\gamma = \rho g$

 $\rho = 750 \text{ kg/m}^3$



$$\rho = \frac{M}{V}$$

i) $\forall = \frac{V}{M}$
 $= \frac{4}{3000}$
 $\forall = 1.33 \times 10^{-3} \text{ m}^3/\text{kg}$
 $\forall = 1.33 \times 10^{-3} \text{ m}^3/\text{kg}$
 $\forall = 1.33 \times 10^{-3} \text{ m}^3/\text{kg}$
 $\forall = 1.33 \times 10^{-3} \text{ m}^3/\text{kg}$

$$=\frac{7357.5}{9810} or S = \frac{750}{1000}$$

S = 0.75 S = 0.75

2. Calculate specific weight, density, specific volume and specific gravity and if one liter of Petrol weighs 6.867N.

 $\gamma = \frac{W}{V}$ $= \frac{6.867}{10^{-3}}$ W = 6.867N $\gamma = 6867N/m^{3}$



$$S = \frac{\gamma}{\gamma_{s \text{ tan dard}}}$$

$$\rho = S g$$

$$= \frac{6867}{9810}$$

$$6867 = \rho \times 9.81$$

$$S = 0.7$$

$$\rho = 700 \text{kg/m}^3$$

$$\forall = \frac{V}{M} \qquad M = W/g$$
$$= \frac{10^{-3}}{0.7} \qquad M = 6.867 \div 9.81$$
$$\forall = 1.4x10^{-3}m^3/kg \qquad M = 0.7 \text{ kg}$$

3. Specific gravity of a liquid is 0.7 Find i) Mass density ii) specific weight. Also find the mass and weight of 10 Liters of liquid.

$$S = \frac{\gamma}{\gamma_{s \tan dard}}$$

$$S = 0.7$$

$$V = ?$$

$$\rho = ?$$

$$0.7 = \frac{\gamma}{9810}$$

$$6867 = \rho \times 9.81$$

$$M = ?$$

$$W = ?$$

$$W = ?$$

$$V = 10 \text{ litre}$$

$$= 10 \times 10^{-3} \text{ m}^{3}$$



$S = \frac{\rho}{\rho_{S \tan dard}}$	$\gamma = \frac{W}{V}$
$0.7 = \frac{\rho}{1000}$	$6867 = \frac{W}{10^{-2}}$
$\rho = 700 \text{kg}/\text{m}^3$	W = 68.67 N
$\rho = \frac{M}{V}$	W = m g
$700 = \frac{M}{10 \times 10^{-3}}$	= 7 x 9.81
M = 7kg	W = 68.67 N

4. **Vapour Pressure:** The process by which the molecules of the liquid go out of its surface in the form of vapour is called Vapourisation.

There are two ways of causing Vapourisation.

- a) By increasing the temperature of the liquid to its boiling point.
- b) By reducing the pressure above the surface of the liquid to a value less than Vapour pressure of the liquid.





As the pressure above the surface of the liquid is reduced, at some point, there will be vapourisation of the liquid. If the reduction in pressure is continued vapourisation will also continue. If the reduction in pressure is stopped, vapourisation continues until vapours of the liquid exert certain pressure which will just stop the vapourisation. This minimum partial pressure exerted by the vapours of the liquid just to stop vapourisation is called <u>Vapour Pressure</u> of the liquid.

If the pressure over the surface goes below the vapour pressure, then, there will be vapourisation. But if the pressure above the surface is more than the vapour pressure then there will not be vapourisation unless there is heating.

• Importance of Vapour Pressure:

- In case of Hydraulic turbines sometimes pressure goes below the vapour pressure of the liquid. This leads to vapourisation and formation of bubbles of liquid. When bubbles are carried to high Pressure zone they get busted leaving partial vacuum. Surrounding liquid enters this space with very high velocity exerting large force on the part of the machinery. This shenornenon is called <u>cavitation</u>. Turbines are designed such that there is no cavitation.
- 2. In Carburetors and sprayers vapours of liquid are created by reducing the pressure below vapour pressure of the liquid.

Unit of Vapour Pressure: N/m² (Pascal - Pa)

Vapour Pressure of a fluid increases with increase in temperature.

Problem

1. A vertical cylinder 300mm in diameter is fitted at the top with a tight but frictionless piston and filled with water at 70° C. The outer portion of the piston is exposed to atmospheric pressure of 101.3 kPa. Calculate the minimum force applied on the piston that will cause water to boil at 70° C. Take Vapour pressure of water at 70° C as 32k Pa.





D = 300 mm= 0.3 m

. . . .

F Should be applied such that the Pressure is reduced from 101.3kPa to 32kPa. There fore reduction in pressure required

= 101.3 - 32
= 69.3 kPa
=69.3 x 10³ N/m²
∴ F / Area = 69.3 x 10³
F /
$$\frac{\Pi}{4}$$
 x (0.3)² = 69.3 x 10³
F = 4.9 x 10³N
F = 4.9 kN

6. Viscosity:

Viscosity is the property by virtue of which fluid offers resistance against the flow or shear deformation. In other words, it is the reluctance of the fluid to flow. Viscous force is that force of resistance offered by a layer of fluid for the motion of another layer over it.

In case of liquids, viscosity is due to cohesive force between the molecules of adjacent layers of liquid. In case of gases, molecular activity between adjacent layers is the cause of viscosity.



• Newton's law of viscosity:

Let us consider a liquid between the fixed plate and the movable plate at a distance 'Y' apart , 'A' is the contact area (Wetted area) of the movable plate , 'F' is the force required to move the plate with a velocity 'U' According to Newton



' μ ' is the constant of proportionality called <u>Dynamic Viscosity</u> or Absolute Viscosity or Coefficient of Viscosity or Viscosity of the fluid.

$$\frac{F}{A} = \mu \cdot \frac{U}{Y}$$
$$\therefore \tau = \mu \frac{U}{Y}$$



' τ ' is the force required; per unit area called 'Shear Stress'.

The above equation is called Newton's law of viscosity.

Velocity gradient or rate of shear strain:

It is the difference in velocity per unit distance between any two layers.

If the velocity profile is linear then velocity gradient is given by $\frac{U}{Y}$. If the velocity profile

is non – linear then it is given by $\frac{du}{dy}$.

- Unit of force (F): N.
- Unit of distance between the tow plates (Y): m
- Unit of velocity (U): m/s

• Unit of velocity gradient :
$$\frac{U}{Y} = \frac{m/s}{m} = /s = s^{-1}$$

• Unit of dynamic viscosity (τ): $\tau = \mu \cdot \frac{u}{y}$

$$\mu = \frac{\tau y}{U}$$
$$\Rightarrow \frac{N/m^2 \cdot m}{m/s}$$
$$\mu \Rightarrow \frac{Ns}{m^2} \text{ or } \mu \Rightarrow P_a s$$

NOTE:

In CGS system unit of dynamic viscosity is $\frac{\text{dyne} \cdot \text{Sec}}{\text{cm}^2}$ and is called poise (P).

If the value of μ is given in poise, multiply it by 0.1 to get it in $\frac{NS}{m^2}$.





• Effect of Pressure on Viscosity of fluids:

Pressure has very little or no effect on the viscosity of fluids.

• Effect of Temperature on Viscosity of fluids:

- 1. *Effect of temperature on viscosity of liquids:* Viscosity of liquids is due to cohesive force between the molecules of adjacent layers. As the temperature increases cohesive force decreases and hence viscosity decreases.
- 2. *Effect of temperature on viscosity of gases:* Viscosity of gases is due to molecular activity between adjacent layers. As the temperature increases molecular activity increases and hence viscosity increases.
- Kinematics Viscosity: It is the ratio of dynamic viscosity of the fluid to its mass density.

$$\therefore \text{ Kinematic Vis cosity} = \frac{\mu}{\rho}$$

Unit of KV:

$$KV \Rightarrow \frac{\mu}{\rho}$$
$$\Rightarrow \frac{NS/m^2}{kg/m^3}$$

$$=\frac{NS}{m^2}x\frac{m^3}{kg}$$

$$= \left(\frac{kg m}{s^2}\right) x \frac{s}{m^2} x \frac{m^3}{kg} = m^2 / s$$

$$N = Kg.m / s^2$$

 \therefore KinematicViscosity = m²/s

NOTE: Unit of kinematics viscosity in CGS system is $\underline{\text{cm}^2/\text{s}}$ and is called $\underline{\text{stoke}(S)}$ If the value of KV is given in stoke, multiply it by 10⁻⁴ to convert it into m²/s.



Problems:

1. Viscosity of water is 0.01 poise. Find its kinematics viscosity if specific gravity is 0.998.

Kinematics viscosity = ?

$$S = 0.998$$

 $S = 0.998$
 $S = \frac{\rho}{\rho_{stan drad}}$
 $\mu = 0.001 \frac{NS}{m^2}$
 $\therefore KV = \frac{\mu}{\rho}$
 $0.998 = \frac{\rho}{1000}$
 $= \frac{0.001}{998}$
 $KV = 1 \times 10^{-6} \text{ m}^2/\text{s}$
 $\rho = 998 \text{ kg/m}^3$

2. A Plate at a distance 0.0254mm from a fixed plate moves at 0.61m/s and requires a force of 1.962N/m² area of plate. Determine dynamic viscosity of liquid between the plates.

$$Y = 0.0254 \text{ mm}$$

= 0.0254 x 10⁻³m
 $\tau = 1.962 \text{ N/m}^2$
 $\mu = ?$

Assuming linear velocity distribution

$$\tau = \mu \ \frac{U}{Y}$$



$$1.962 = \mu x \frac{0.61}{0.0254 x 10^{-3}}$$
$$\mu = 8.17 x 10^{-5} \frac{\text{NS}}{\text{m}^2}$$

3. A plate having an area of $1m^2$ is dragged down an inclined plane at 45^0 to horizontal with a velocity of 0.5m/s due to its own weight. Three is a cushion of liquid 1mm thick between the inclined plane and the plate. If viscosity of oil is 0.1 Pas find the weight of the plate.

y = 1 mm = 1 x 10⁻³ m
Plate
U = 0.5 m/s
Y = 1 x 10⁻³ m

$$\mu = 0.1 \text{NS/m}^2$$

W = ?
F = W x cos 45⁰
= W x 0.707
F = 0.707W
 $\tau = \frac{F}{A}$
 $\tau = 0.707 \text{ W N/m}^2$



Assuming linear velocity distribution,

$$\tau = \mu \cdot \frac{U}{Y}$$

0.707W = 0.1 x $\frac{0.5}{1 \text{ x } 10^{-3}}$
W = 70.72 N

A shaft of φ 20mm and mass 15kg slides vertically in a sleeve with a velocity of 5 m/s. The gap between the shaft and the sleeve is 0.1mm and is filled with oil. Calculate the viscosity of oil if the length of the shaft is 500mm.



 $D = 20mm = 20x10^{-3}m$

M = 15 kgW = 15x 9.81W = 147.15N

y = 0.1mm $y = 0.1 \times 10^{-3}$ mm



U = 5m/sF = WF = 147.15N $\mu = ?$ $A = \Pi D L$ $A = \Pi x 20 x 10^{-3} x 0.5$ $A = 0.031 \text{ m}^2$ $\tau = \mu . \frac{U}{V}$ $4746.7 = \mu x \frac{5}{0.1x10^{-3}}$ $\mu = 0.095 \ \frac{NS}{m^2}$ $\tau = \frac{F}{A}$ $=\frac{147.15}{0.031}$ $\tau = 4746.7 \,\text{N} \,/\,\text{m}^2$

- 5. If the equation of velocity profile over 2 plate is $V=2y^{2/3}$. in which 'V' is the velocity in m/s and 'y' is the distance in 'm'. Determine shear stress at (i) y = 0 (ii) y = 75mm. Take $\mu = 8.35$ P.
 - (i) at y = 0
 - (ii) at y = 75mm = 75 x 10^{-3} m



 $\tau = 8.35 P$ $= 8.35 \ge 0.1 \frac{NS}{m^2}$ $= 0.835 \frac{NS}{m^2}$ $V = 2y^{2/3}$ $\frac{dv}{dy} = 2x\frac{2}{3}y^{2/3-1}$ $=\frac{4}{3}y^{-1/3}=\frac{4}{3}\frac{1}{\sqrt[3]{y}}$ at, y = 0, $\frac{dv}{dy} = 3\frac{4}{\sqrt[3]{0}} = \infty$ at, y = 75×10^{-3} m, $\frac{dv}{dy} = 3 \frac{4}{\sqrt[3]{75 \times 10^{-3}}}$ $\frac{dv}{dy} = 3.16/s$ $\tau = \mu . \frac{dv}{dv}$ *at*, y = 0, $\tau = 0.835x\infty$ $\tau = \infty$

 $at, y = 75 x 10^{-3} m, \tau = 0.835 x 3.16$

$$\tau = 2.64 \ N \,/\, m^2$$



6. A circular disc of 0.3m dia and weight 50 N is kept on an inclined surface with a slope of 45⁰. The space between the disc and the surface is 2 mm and is filled with oil of dynamics viscosity $\frac{1NS}{m^2}$. What force will be required to pull the disc up the inclined plane with a velocity of 0.5m/s.



$$D = 0.3m$$

$$A = \frac{\prod x \ 0.3m^2}{4}$$

$$\frac{A = 0.07m^2}{4}$$

$$W = 50N$$

$$\mu = 1 \frac{NS}{m^2}$$

$$F = P - 50 \cos 45$$

$$F = (P - 35,35)$$

$$\frac{y = 2x10^{-3}m}{U = 0.5m/s}$$

$$v = \frac{(P - 35,35)}{0.07} N/m^2$$

$$\tau = \mu . \frac{U}{Y}$$

$$\left(\frac{P - 35,35}{0.07}\right) = 1x \frac{0.5}{2x10^{-3}}$$

P = 52.85N



7. Dynamic viscosity of oil used for lubrication between a shaft and a sleeve is 6 P. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 0.09 m. Thickness of oil is 1.5 mm.





$$U = \frac{\Pi DN}{60}$$

= $\frac{\Pi \ge 0.4 \ge 190}{60}$
U = 3.979 m/s
 $\tau = \mu \cdot \frac{U}{Y}$
= $0.6 \ge \frac{3.979}{1.5 \ge 10^{-3}}$
 $\tau = 1.592 \ge 10^{3} \text{ N/m}^{2}$
 $\frac{F}{A} = 1.59 \ge 10^{3}$
F = $1.591 \ge 10^{3} \ge 0.11$
F = $175.01 \ge N$
T = F $\ge R$
= $175.01 \ge 0.2$
T = $35 \ge 10^{3}$
P = $\frac{2\Pi NT}{60,000}$
P = $0.6964 \le 10^{3}$



8. Two large surfaces are 2.5 cm apart. This space is filled with glycerin of absolute viscosity 0.82 NS/m². Find what force is required to drag a plate of area 0.5m² between the two surfaces at a speed of 0.6m/s. (i) When the plate is equidistant from the surfaces, (ii) when the plate is at 1cm from one of the surfaces.

Case (i)



Let F_1 be the force required to overcome viscosity resistance of liquid above the plate and F_2 be the force required to overcome viscous resistance of liquid below the plate. In this case $F_1 = F_2$. Since the liquid is same on either side or the plate is equidistant from the surfaces.

$$\tau_{1} = \mu_{1} \frac{U}{Y}$$

$$\tau_{1} = 0.82 \times \frac{0.6}{0.0125}$$

$$\tau_{1} = 39.36 \text{ N/m}^{2}$$

$$\frac{F_{1}}{A} = 39.36$$

$$F_{1} = 19.68 \text{ N}$$

 \therefore Tatal force required to drag the plate =F₁ +F₂ = 19.68+19.68

F= 39.36N



Case (ii)

2.5 cm
$$F_1$$
 0.6 m/s F_2 F_2

Here $F_1 \neq F_2$

$$\tau_1 = \mu_1 \frac{U}{y1}$$

$$= 0.82 \text{ x} \frac{0.62}{1 \text{ x} 10^{-2}}$$

$$\tau_1 = 49.2 \text{ N/m}^2$$

$$\tau_2 = \mu_2 \frac{U}{y_2}$$

$$= 0.82 \text{ x} \frac{0.6}{1.5 \text{ x} 10^{-2}}$$

$$\tau_1 = 32.8 \text{ N/m}^2$$



$$\frac{F_1}{A} = 49.2$$

$$F_1 = 49.2 \times 0.5$$

$$F_1 = 24.6 \text{N}$$

$$\frac{F_2}{A} = 32.8$$

$$F_2 = 32.8 \times 0.5$$

$$F_2 = 16.4 \text{N}$$

Total Force $F = F_1 + F_2 = 24.6 + 16.4$ F = 41N

- 9. Through a very narrow gap of height a thin plate of large extent is pulled at a velocity V'. On one side of the plate is oil of viscosity μ_1 and on the other side there is oil of viscosity μ_2 . Determine the position of the plate for the following conditions.
 - i. Shear stress on the two sides of the plate is equal.
 - ii. The pull required, to drag the plate is minimum.

Conditions 1





$$y = ? \text{ for } F_1 = F_2$$

$$\tau = \mu \cdot \frac{U}{Y}$$

$$\frac{F}{A} = \mu \cdot \frac{U}{Y}$$

$$F = A\mu \cdot \frac{U}{Y}$$

$$F_1 = \frac{A\mu_1 V}{(h - y)}$$

$$F_2 = \frac{A\mu_2 V}{y}$$

$$F_1 = F_2$$

$$\frac{A\mu_1 V}{h - y} = \frac{A\mu_2 V}{y}$$

$$\mu_1 y = \mu_2 (h - y)$$

$$\mu_1 y + \mu_2 y = \mu_2 h$$

$$y = \frac{\mu_2 h}{\mu_1 + \mu_2} \text{ or } y = \frac{h}{\mu_1} + 1$$



Conditions 2:



y = ? if, $F_1 + F_2$ is to be minimum

$$\mathrm{F}_1 {=} \frac{\mathrm{A} \mu_1 \mathrm{V}}{\mathrm{h-y}}$$

$$F_2 {=} \frac{A\mu_2 V}{y}$$

: Total drag forced required



$$F = F_{1} + F_{2}$$

$$F = \frac{A\mu_{1}V}{h-y} + \frac{A\mu_{2}V}{y}$$
For F to be min. $\frac{dF}{dy} = 0$

$$\frac{dF}{dy} = 0 = +A\mu_{1}V \equiv (h-y)^{-2} - A\mu_{2}Vy^{-2}$$

$$= \frac{V\mu_{1}A}{(h-y)^{+2}} - \frac{V\mu_{2}A}{y^{2}}$$

$$\frac{(h-y)^{2}}{y^{2}} = \frac{\mu_{1}}{\mu_{2}}$$

$$\frac{h-y}{y} = \sqrt{\frac{\mu_{1}}{\mu_{2}}}$$

$$(h-y) = y\sqrt{\frac{\mu_{1}}{\mu_{2}}}$$

$$h = y\sqrt{\frac{\mu_{1}}{\mu_{2}}} + y$$

$$h = y\left(1 + \sqrt{\frac{\mu_{1}}{\mu_{2}}}\right)$$

$$\therefore y = \frac{h}{1+\sqrt{\frac{\mu_{1}}{\mu_{2}}}}$$



(7) Surface Tension (σ)



Surface tension is due to cohesion between the molecules of liquid and weak adhesion between the molecules on the exposed surface of the liquid and molecules of air.

A molecule inside the surface gets attracted by equal forces from the surrounding molecules whereas a molecule on the surface gets attracted by the molecule below it. Since there are no molecules above it, it experiences an unbalanced vertically downward force. Due to this entire surface of the liquid expose of to air will have a tendency to move in ward and hence the surface will be under tension. The property of the liquid surface to offer resistance against tension is called surface tension.

• Consequences of Surface tension:

- Liquid surface supports small loads.
- Formation of spherical droplets of liquid
- Formation of spherical bubbles of liquid
- Formation of cylindrical jet of liquids.



- F
- Measurement of surface tension:

Surface tension is measured as the force exerted by the film on a line of unit length on the surface of the liquid. It can also be defined as the force required maintaining unit length of film in equilibrium.

$$\therefore \sigma = \frac{F}{L} \qquad \qquad \therefore F = \sigma L$$

Unit: N/m

Force due to surface tension = σ x length of film

NOTE: Force experienced by a curved surface due to radial pressure is given by the product of intensity of pressure and projected area of the curved surface.







• To derive an expression for the pressure inside the droplet of a liquid.



Let us consider droplet of liquid of surface tension ' σ '. 'D' is the diameter of the droplet. Let 'p' be the pressure inside the droplet in excess of outside pressure $(p = p_{inside} - p_{outside})$.

For the equilibrium of the part of the droplet,

Force due to surface ten	sion	=	Force due to pressure
σxΠD		=	p x projected area
σxΠD		=	$p x \frac{\Pi D^2}{4}$
p	þ	=	$\frac{4\sigma}{D}$

As the diameter increases pressure decreases.



• To derive an expression for the pressure inside the bubble of liquid:

`D' is the diameter of bubble of liquid of surface tension σ . Let 'p' be the pressure inside the bubble which is in excess of outside pressure. In case of bubble the liquid layer will be in contact with air both inside and outside.



For the equilibrium of the part of the bubble,

Force due to surface tension = Force due to pressure

 $(2 \sigma) \times \Pi D = p \times \text{projected area}$

$$2[\sigma x \Pi D] = p x \frac{\Pi D^2}{4}$$
$$p = \frac{8\sigma}{D}$$

• To derive an expression for the pressure inside the jet of liquid:



Let us consider a jet of diameter D of liquid of surface tension σ and p is the intensity of pressure inside the jet in excess of outside atmospheric pressure. For the equilibrium of the part of the jet shown in fig,


Force due to Radial pressure = Force due to surface tension

p x Projected area = σ x Length p x D x L = σ x 2L $P = \frac{2\sigma}{D}$

4σ

• Effect of temperature on surface tension of liquids:

In case of liquids, surface tension decreases with increase in temperature. Pressure has no or very little effect on surface tension of liquids.

Problems:

1. What is the pressure inside the droplet of water 0.05mm in diameter at 20^oC, if the pressure outside the droplet is 103 kPa Take $\sigma = 0.0736$ N/m at 20^oC

$$p = \frac{1}{D}$$

$$= \frac{4 \times 0.0736}{0.05 \times 10^{-3}}$$

$$p = 5.888 \times 10^{+3} \text{ N/m}^2$$

$$p = p_{\text{inside}} - p_{\text{outside}}$$

$$p = p_{\text{inside}} - p_{\text{outside}}$$

$$p_{\text{inside}} = (5.888 + 103)10^3$$

$$\sigma = 0.0736 \text{ N/m}$$

$$p_{\text{inside}} = 108.88 \times 10^3 \text{ Pa}$$



2. A liquid bubble 2cm in radius has an internal pressure of 13Pa. Calculate the surface tension of liquid film.

$$p = \frac{8\sigma}{D} \qquad R = 2cm$$

$$\sigma = \frac{13x4x10^{-2}}{8} \qquad D = 4cm$$

$$\sigma = 0.065N/m \qquad p = 13Pa(N/m^2)$$

8. Capillarity:



Any liquid between contact surfaces attains curved surface as shown in figure. The curved surface of the liquid is called Meniscus. If adhesion is more than cohesion then the meniscus will be concave. If cohesion is greater than adhesion meniscus will be convex.







Capillarity is the phenomena by which liquids will rise or fall in a tube of small diameter dipped in them. Capillarity is due to cohesion / adhesion and surface tension of liquids. If adhesion is more than cohesion then there will be capillary rise. If cohesion is greater than adhesion then will be capillary fall or depression. The surface tensile force supports capillary rise or depression.

Note:





The angle between surface tensile force and the vertical is called angle of contact. If adhesion is more than cohesion then angle of contact is obtuse.



• To derive an expression for the capillary rise of a liquid in small tube dipped in it:

Let us consider a small tube of diameter 'D' dipped in a liquid of specific weight γ . 'h' is the capillary rise. For the equilibrium,

Vertical force due to surface tension = Weight of column of liquid ABCD



$$[\sigma(\Pi D)] \cos \theta = \gamma x$$
 volume

$$\left[\sigma(\Pi D)\right]\cos\theta = \gamma x \frac{\Pi D^2}{4} x h$$
$$h = \frac{4\sigma\cos\theta}{4}$$

$$=$$
 γD

h

It can be observed that the capillary rise is inversely proportional to the diameter of the tube.

Note:

The same equation can be used to calculate capillary depression. In such cases ' θ ' will be obtuse 'h' works out to be –ve.



Problems:

1. Capillary tube having an inside diameter 5mm is dipped in water at 20⁰. Determine the heat of water which will rise in tube. Take $\sigma = 0.0736$ N/m at 20⁰ C.

$$h = \frac{4 \sigma \cos \theta}{\gamma D}$$

$$= \frac{4 x \ 0.0736 \ x \ \cos \theta}{9810 \ x \ 5 \ x \ 10^{-3}}$$

$$\theta = 0^{0} \text{(assumed)}$$

$$\gamma = 9810 \text{N/m}^{3}$$

$$h = 6 \ x \ 10^{-3} \text{m}$$

2. Calculate capillary rise in a glass tube when immersed in Hg at 20^oc. Assume σ for Hg at 20^oc as 0.51N/m. The diameter of the tube is 5mm. $\theta = 130^{\circ}c$.

$$h = \frac{4 \sigma \cos \theta}{\gamma D}$$

$$h = -1.965 \text{ x} 10^{-3} \text{ m}$$

$$S = \frac{\gamma}{\gamma_{\text{S tan dard}}}$$

$$13.6 = \frac{\gamma}{9810}$$

$$\gamma = 133.416 \text{ x} 10^3 \text{ N/m}^3$$

-ve sign indicates capillary depression.



3. Determine the minimum size of the glass tubing that can be used to measure water level if capillary rise is not to exceed 2.5mm. Take $\sigma = 0.0736$ N/m.

$$h = \frac{4\sigma \cos \theta}{\gamma D}$$

$$D = \frac{4 \times 0.0736 \times \cos 0}{9810 \times 2.5 \times 10^{-3}}$$

$$D = ?$$

$$h = 2.5 \times 10^{-3} m$$

$$D = 0.012 m$$

$$\sigma = 0.0736 \text{ N/m}$$

$$D = 12 \text{ mm}$$

4. A glass tube 0.25mm in diameter contains Hg column with air above it. If $\sigma = 0.51$ N/m, what will be the capillary depression? Take $\theta = -40^{\circ}$ or 140° .

$h = \frac{4\sigma\cos\theta}{\gamma D}$	$D = 0.25 \times 10^{-3} m$
4x0.51x cos140	$\sigma = 0.51 \text{N/m}$
$=\frac{133.146 \times 10^{-3} \times 0.25 \times 10^{-3}}{133.146 \times 10^{-3} \times 0.25 \times 10^{-3}}$	$\theta = 140$
$h = -46.851 x 10^{-3} m$	$\gamma = 133.416 \text{ x } 10^3 \text{ N/m}^2$



5. If a tube is made so that one limb is 20mm in ϕ and the other 2mm in ϕ and water is poured in the tube, what is the difference in the level of surface of liquid in the two limbs. $\sigma = 0.073$ N/m for water.





6. Compressibility:

It is the property by virtue of which there will be change in volume of fluid due to change in pressure.

Let 'v' be the original volume and 'dv' be the change in volume due to change in pressure 'dp', $\frac{dv}{v}$ i.e., the ratio of change in volume to original volume is called volumetric strain or bulk strain.

The ratio of change in pressure to the volumetric strain produced is called Bulk modulus of elasticity of the fluid and is denoted by 'K'

$$\therefore K = \frac{dp}{\left(\frac{dv}{v}\right)}.$$

Sometimes 'K' is written as $K = -\frac{dp}{\left(\frac{dv}{v}\right)}$. -ve sign indicates that as there is

increase in pressure, there is decrease in volume. Reciprocal of Bulk modulus of elasticity is called Compressibility of the fluid.

$$\therefore \text{ Compressibility} = \frac{1}{K} = \frac{\frac{dv}{v}}{dp}$$

Unit of Bulk modulus of elasticity is N/m^2 or Pa. Unit of compressibility is m^2/N .



Problem:

1. The change in volume of certain mass of liquids is observed to be $\frac{1}{500}$ th of original volume when pressure on it is increased by 5Mpa. Determine the Bulk modulus and compressibility of the liquid.

$$dv = \frac{1}{500}V$$

$$K = \frac{dp}{dv}$$

$$\frac{dv}{v} = \frac{1}{500}$$

$$= 2.5 \times 10^{9} Pa$$

$$dp = 5 \times 10^{6} N/m^{2}$$

$$K = 2.5 GPa$$

$$K = 2.5 GPa$$

$$Compresibility = \frac{1}{K}$$

$$= \frac{1}{25 \times 10^{8}}$$

$$= 4 \times 10^{-10} m^{2}/N$$

2. Find the pressure that must be applied to water at atmospheric pressure to reduce its volume by 1% .Take K=2 GPa.

$$K = \frac{dp}{dv}$$

$$2 \times 10^9 = \frac{dP}{\frac{1}{100}}$$

$$dp = 20 \times 10^6 \text{ N/m}^2$$

$$dp = 20 \text{ MPa}$$



• Rheological classification of fluids: (Rheology → Study of stress – strain behavior).

1. Newtonian fluids: A fluid which obeys Newton's law of viscosity i.e., $\tau = \mu$. $\frac{du}{dy}$ is called Newtonian fluid. In such fluids shear stress varies directly as shear strain.

In this case the stress strain curve is a stress line passing through origin the slope of the line gives dynamic viscosity of the fluid.

Eg: Water, Kerosene.





2. **Non- Newtonian fluid:** A fluid which does not obey Newton's law of viscosity is called non-Newton fluid. For such fluids,



3. Ideal Plastic fluids:

In this case the strain starts after certain initial stress (τ_0) and then the stressstrain relationship will be linear. τ_0 is called initial yield stress. Sometimes they are also called <u>Bingham's Plastics:</u>

Eg: Industrial sludge.





4. Thixotropic fluids:

These require certain amount of yield stress to initiate shear strain. After wards stress-strain relationship will be non – linear.

Eg; Printers ink.



5. Ideal fluid:

Any fluid for which viscosity is assumed to be zero is called Ideal fluid. For ideal





6. Real fluid :

Any fluid which posses certain viscosity is called real fluid. It can be Newtonian or non – Newtonian, thixotropic or ideal plastic.







Unit-II: PRESSURE AND ITS MEASUREMENTS

Fluid is a state of matter which exhibits the property of flow. When a certain mass of fluids is held in static equilibrium by confining it within solid boundaries, it exerts force along direction perpendicular to the boundary in contact. This force is called fluid pressure.

• Pressure distribution:

It is the variation of pressure over the boundary in contact with the fluid.

There are two types of pressure distribution.

- a) Uniform Pressure distribution.
- b) Non-Uniform Pressure distribution.

(a) Uniform Pressure distribution:



If the force exerted by the fluid is same at all the points of contact boundary then the pressure distribution is said to be uniform.

(b) Non –Uniform Pressure distribution:



If the force exerted by the fluid is not same at all the points then the pressure distribution is said to be non-uniform.

• Intensity of pressure or unit pressure or Pressure:

Intensity of pressure at a point is defined as the force exerted over unit area considered around that point. If the pressure distribution is uniform then intensity of pressure will be same at all the points.

• Calculation of Intensity of Pressure:

When the pressure distribution is uniform, intensity of pressure at any points is given by the ratio of total force to the total area of the boundary in contact.

:. Intensity of Pressure 'p' = $\frac{F}{A}$

When the pressure distribution is non- uniform, then intensity of pressure at a point is given by $\frac{dF}{dA}$.

• Unit of Intensity of Pressure:

N/m² or pascal (Pa).

Note: 1 MPa =1N/mm²



• To study the variation of intensity of pressure in a static mass of fluid: or derive hydrostatic law of pressure.



Self weight = γx volume = $\gamma x dx dy dz$

Let us consider a point 'M' at a depth 'y' below the free surface of the liquid of specific weight ' γ '. (dx dy dz) is the elemental volume of the fluid considered around the point 'M'.

Fig shows forces acting on the element including self weight. The element of fluid is in equilibrium and hence we can apply conditions of equilibrium.



$$\sum Fx=0$$

$$+\left[p-\frac{\partial p}{\partial x}\cdot\frac{dx}{2}\right]dy \ dz - \left[P+\frac{\partial p}{\partial x}\cdot\frac{dx}{2}\right]dy \ dz = 0$$

$$i, e \ p-\frac{\partial p}{\partial x}\cdot\frac{dx}{2} - p - \frac{\partial p}{\partial x}\cdot\frac{dx}{2} = 0$$

$$-2\cdot\frac{\partial P}{\partial x}\cdot\frac{dx}{2} = 0$$

$$\therefore \frac{\partial P}{\partial x} = 0$$

 \therefore Rate of change of intensity of pressure along x – direction is zero. In other words there is no change in intensity of pressure along x – direction inside the fluid.

$$\sum Fz = 0$$

+ $\left[p - \frac{\partial p}{\partial z} \cdot \frac{dz}{2} \right] dx \quad dy - \left[p + \frac{\partial p}{\partial z} \cdot \frac{dz}{2} = 0 \right] dx \quad dy = 0$
$$\therefore \frac{\partial p}{\partial z} = 0$$

:. The Rate of change of intensity of pressure along z direction is zero.

In other words there is no change in intensity of pressure along z – direction.



$$\sum Fy=0$$

$$+\left[p - \frac{\partial p}{\partial y} \cdot \frac{dy}{2}\right] dx dz - \left[p + \frac{\partial p}{\partial y} \cdot \frac{dy}{2}\right] dx dz - \gamma dx dy dz = 0$$
i.e. $p - \frac{\partial p}{\partial y} \cdot \frac{dy}{2} - p - \frac{\partial P}{\partial y} \cdot \frac{dy}{2} = \gamma dy$

$$i, e - \frac{\partial p}{\partial y} dy = \gamma dy$$

$$\therefore \frac{\partial p}{\partial y} = -\gamma$$

-ve sign indicates that the pressure increases in the downward direction i.e., as the depth below the surface increases intensity of pressure increases.

$$\therefore \frac{\partial p}{\partial y} = \gamma$$

$$\therefore \partial p = \gamma . \text{ oy}$$

integrating,
$$p = \gamma y + C$$

at y = 0; p = p_{\text{Atmospheric}}
$$p_{\text{atm}} = \gamma x0 + C$$

$$\therefore C = p_{\text{atm}}$$

$$\therefore p = \gamma y + p_{\text{atm}}$$

The above equation is called hydrostatic law of pressure.



• Atmospheric pressure

Air above the surface of liquids exerts pressure on the exposed surface of the liquid and normal to the surface.

This pressure exerted by the atmosphere is called atmospheric pressure. Atmospheric pressure at a place depends on the elevation of the place and the temperature.

Atmospheric pressure is measured using an instrument called 'Barometer' and hence atmospheric pressure is also called Barometric pressure.

Unit: kPa.

'bar' is also a unit of atmospheric pressure 1bar = 100 kPa.

• Absolute pressure and Gauge Pressure:



Absolute zero pressure line

Absolute pressure at a point is the intensity of pressure at that point measured with reference to absolute vacuum or absolute zero pressure.

Absolute pressure at a point can never be negative since there can be no pressure less than absolute zero pressure.



If the intensity of pressure at a point is measured with reference to atmospheric pressure, then it is called gauge pressure at that point.

Gauge pressure at a point may be more than the atmospheric pressure or less than the atmospheric pressure. Accordingly gauge pressure at the point may be positive or negative.

Negative gauge pressure is also called vacuum pressure.

From the figure, It is evident that, Absolute pressure at a point = Atmospheric pressure \pm Gauge pressure.

NOTE: If we measure absolute pressure at a Point below the free surface of the liquid, then,

$$p = \gamma$$
. Y + p_{atm}

If gauge pressure at a point is required, then atmospheric pressure is taken as zero, then,

 $p = \gamma$. Y

Pressure Head

It is the depth below the free surface of liquid at which the required pressure intensity is available.

 $P = \gamma h$

$$\therefore h = \frac{P}{\gamma}$$

For a given pressure intensity 'h' will be different for different liquids since, ' γ ' will be different for different liquids.



: Whenever pressure head is given, liquid or the property of liquid like specify gravity, specific weight, mass density should be given.

Eg:

- (i) 3m of water
- (ii) 10m of oil of S = 0.8.
- (iii) 3m of liquid of $\gamma = 15 \text{ kN/m}^3$
- (iv) 760mm of Mercury.
- (v) $10m \rightarrow \text{not correct.}$

NOTE:

1. To convert head of a liquid to head of another liquid.

$$\begin{split} S &= \frac{\gamma}{\gamma_{\text{Standard}}} \\ S_1 &= \frac{\gamma_1}{\gamma_{\text{Standard}}} \\ p &= \gamma_1 h_1 \\ p &= \gamma_2 h_2 \\ \hline \gamma_1 h_1 &= \gamma_2 h_2 \\ \hline \gamma_1 h_1 &= \gamma_2 h_2 \\ \hline \ddots S_1 \gamma_{\text{Standard}} h_1 &= S_2 \gamma_{\text{Standard}} h_2 \\ \hline S_1 h_1 &= S_2 h_2 \\ \hline \end{split}$$



2. $S_{water} \ge h_{water} = S_{liquid} \ge h_{liquid}$ $1 \ge h_{water} = S_{liquid} \ge h_{liquid}$ $h_{water} = S_{liquid} \ge h_{liquid}$

Pressure head in meters of water is given by the product of pressure head in meters of liquid and specific gravity of the liquid.

Eg: 10meters of oil of specific gravity 0.8 is equal to 10x0.8 = 8 meters of water.

Eg: Atmospheric pressure is 760mm of Mercury.

NOTE:

$$P = \gamma \quad h$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$kPa \quad \frac{kN}{m^3} \quad m$$

Problem:

- 1. Calculate intensity of pressure due to a column of 0.3m of (a) water (b) Mercury
 - (c) Oil of specific gravity-0.8.

a) h = 0.3m of water

$$\gamma = 9.81 \frac{\text{kN}}{\text{m}^3}$$

$$p = ?$$

$$p = \gamma h$$

$$p = 2.943 \text{ kPa}$$

$$c) h = 0.3 \text{ of Hg}$$

$$\gamma = 13.6 \text{ x } 9.81$$



$$\gamma = 133.416 \text{ kN/m}^3$$

 $p = \gamma \text{ h}$
 $= 133.416 \text{ x } 0.3$
 $p = 40.025 \text{ kPa}$

2. Intensity of pressure required at a points is 40kPa. Find corresponding head in (a) water (b) Mercury (c) oil of specific gravity-0.9.

(a) p = 40 kPa	$h = \frac{p}{\gamma}$ h = 4.077 m of water
$\gamma = 9.81 \frac{kN}{m^3}$	
h = ?	
(b) $p = 40 \text{ kPa}$	
$\gamma = (13.6 \text{x} 9.81 \text{ N/m}^3)$	
$\gamma = 133.416 \ \frac{KN}{m^3}$	$h = \frac{p}{\gamma}$ h = 0.299 m of Mercury

 $h = \frac{p}{\gamma}$

(c) p = 40 kPa

$$h = 4.53 m \text{ of oil } S = 0.9$$

$$\gamma = 0.9 \times 9.81$$
$$\gamma = 8.829 \frac{KN}{m^3}$$



- 3. Standard atmospheric pressure is 101.3 kPa Find the pressure head in (i) Meters of water (ii) mm of mercury (iii) m of oil of specific gravity 0.8.
 - (i) $p = \gamma h$ 101.3 = 9.81 x h h = 10.3 m of water(ii) $p = \gamma h$ 101.3 = (13.6x9.81) x h h = 0.76 m of mercury(iii) $p = \gamma h$ 101.3 = (0.8 x 9.81 x h)

h = 12.9 m of oil of S = 0.8

4. An open container has water to a depth of 2m and above this an oil of S = 0.9 for a depth of 1m. Find the intensity of pressure at the interface of two liquids and at the bottom of the tank.





60

 $p_A = \gamma_{a1} h_{oil}$ = (0.9 x 9.81) x 1 $p_A = 8.829 \text{ kPa}$ $p_B = \gamma_{oil} xh_{oil} + \gamma_{water} + h_{water}$ $p_A = 8.829 \text{ kPa} + 9.81 \text{ x } 2$ $p_B = 28.45 \text{ kPa}$

Convert the following absolute pressure to gauge pressure (a) 120kPa (b) 3kPa (c) 15m of H₂o (d) 800mm of Hg.

(a) p_{abs} = p_{atm} + p_{gauge}
∴ p_{gauge} = p_{abs} - p_{atm} = 120 - 101.3 = 18.7 kPa
(b) p_{gauge} = 3-101.3 = -98.3 kPa
p_{gauge} = 98.3 kPa (vacuum)
(c) h_{abs} = h_{atm} + h_{gauge}
15 =10.3 + h_{gauge}
h_{gauge} = 4.7m of water
(d) h_{abs} = h_{atm} + h_{gauge}
800 = 760 + h_{gauge}
h_{gauge} = 40 mm of mercury



♦ PASCAL 'S LAW

Statement: Intensity of pressure at a point in a static mass of fluid is same along the directions.

Proof:



Let us consider three planes around a point as shown in figure. Figure shows intensity of pressure and force along different directions. The system of forces should be in equilibrium.

$$\therefore \sum Fx = 0$$

$$- p_x dy.dz + p_s ds dz \cos (90^0) = 0$$

$$p_s ds \sin\theta = p_x dy$$

$$p_s dy = p_x dy$$

$$p_s = p_x$$

$$\therefore \sum Fy=0$$

$$- p_s ds.dz \cos \theta + p_y dx dz = 0$$

$$p_y dx = p_s ds \cos \theta$$

$$p_y dx = p_s dx$$

$$p_y = p_s$$

$$\therefore p_x = p_y = p_z$$

:. Intensity of pressure at a point is same along all the directions.



♦ Measurement of Pressure

Various devices used to measure fluid pressure can be classified into,

- 1. Manometers
- 2. Mechanical gauges.

Manometers are the pressure measuring devices which are based on the principal of balancing the column of the liquids whose pressure is to be measured by the same liquid or another liquid.

Mechanical gauges consist of an elastic element which deflects under the action of applied pressure and this movement will operate a pointer on a graduated scale.

Classification of Manometers:

Manometers are broadly classified into

- a) Simple Manometers
- b) Differential Manometers.

a) Simple Manometers

Simple monometers are used to measure intensity of pressure at a point. They are connected to the point at which the intensity of pressure is required. Such a point is called gauge point.

b) Differential Manometers

Differential manometers are used to measure the pressure difference between two points. They are connected to the two points between which the intensity of pressure is required.



♦ Types of Simple Manometers

Common types of simple manometers are

- a) Piezometers
- b) U-tube manometers
- c) Single tube manometers
- d) Inclined tube manometers

a) Piezometers



Piezometer consists of a glass tube inserted in the wall of the vessel or pipe at the level of point at which the intensity of pressure is to be measured. The other end of the piezometer is exposed to air. The height of the liquid in the piezometer gives the pressure head from which the intensity of pressure can be calculated.

To minimize capillary rise effects the diameters of the tube is kept more than 12mm.

Merits

- Simple in construction
- Economical



Demerits

- Not suitable for high pressure intensity.
- Pressure of gases cannot be measured.

(b) U-tube Manometers:



A U-tube manometers consists of a glass tube bent in U-Shape, one end of which is connected to gauge point and the other end is exposed to atmosphere. U-tube consists of a liquid of specific of gravity other than that of fluid whose pressure intensity is to be measured and is called monometric liquid.



• Manometric liquids

- Manometric liquids should neither mix nor have any chemical reaction with the fluid whose pressure intensity is to be measured.
- It should not undergo any thermal variation.
- Manometric liquid should have very low vapour pressure.
- Manometric liquid should have pressure sensitivity depending upon the magnitude of pressure to be measured and accuracy requirement.

• To write the gauge equation for manometers

Gauge equations are written for the system to solve for unknown quantities.

Steps:

- 1. Convert all given pressure to meters of water and assume unknown pressure in meters of waters.
- 2. Starting from one end move towards the other observing the following points.
 - Any horizontal movement inside the same liquid will not cause change in pressure.
 - Vertically downward movement causes increase in pressure and upward motion causes decrease in pressure.
 - Convert all vertical columns of liquids to meters of water by multiplying them by corresponding specify gravity.
 - Take atmospheric pressure as zero (gauge pressure computation).
- 3. Solve for the unknown quantity and convert it into the required unit.
- 4. If required calculate absolute pressure.



Problem:

1. Determine the pressure at A for the U- tube manometer shown in fig. Also calculate the absolute pressure at A in kPa.



Let ' h_A ' be the pressure head at 'A' in 'meters of water'.

 $h_A + 0.75 - 0.5 x 13.6 = 0$

 $h_A = 6.05 m of water$

 $p=\gamma h$

= 9.81 x 6.05

 $p = 59.35 \ kPa(gauge \ pressure)$

$$p_{abs} = p_{atm} + p_{gauge}$$

= 101.3 + 59.35

$$p_{abc} = 160.65 \ kPa$$



2. For the arrangement shown in figure, determine gauge and absolute pressure at the point M.



Let ' h_M ' be the pressure head at the point 'M' in m of water,

 h_M - 0.75 x 0.8 - 0.25 x 13.6 = 0

 $h_{M=4}$ m of water

 $p = \gamma h$

p = 39.24 kPa

 $p_{abs} = 101.3 + 39.24$

 p_{abs} 140.54 kPa

 $p_A = -10 \text{ kPa}$

3. If the pressure at 'At' is 10 kPa (Vacuum) what is the value of 'x'?





$$\frac{p_A}{\gamma} = \frac{-10}{9.81} = -1.019 \text{ m of water}$$
$$h_A = -1.019 \text{ m of water}$$
$$-1.019 + 0.2 \text{ x} 1.2 + \text{ x} (13.6) = 0$$
$$\text{x} = 0.0572 \text{ m}$$

4. The tank in the accompanying figure consists of oil of S = 0.75. Determine the pressure gauge reading in $\frac{kN}{m^2}$.



Let the pressure gauge reading be 'h' m of water

 $h - 3.75 \times 0.75 + 0.25 \times 13.6 = 0$ h = -0.5875 m of water $p = \gamma h$ p = -5.763 kPap = 5.763 kPa (Vacuum)

5. A closed tank is 8m high. It is filled with Glycerine up to a depth of 3.5m and linseed oil to another 2.5m. The remaining space is filled with air under a pressure of 150 kPa. If a pressure gauge is fixed at the bottom of the tank what will be its reading. Also calculate absolute pressure. Take relative density of Glycerine and Linseed oil as 1.25 and 0.93 respectively.





 $P_{\rm H} = 150 \,\text{kPa}$ $h_{\rm M} = \frac{150}{9.81}$ $h_{\rm M} = 15.29 \,\text{m of water}$

Let ' h_N ' be the pressure gauge reading in m of water.

$$h_N$$
 -3.5 x 1.25 -2.5 x 0.93 =15.29
 h_N = 21.99 m of water
 $p = 9.81$ x 21.99
 $p = 215.72$ kPa (gauge)
 $p_{abs} = 317.02$ kPa

6. A vertical pipe line attached with a gauge and a manometer contains oil and Mercury as shown in figure. The manometer is opened to atmosphere. What is the gauge reading at 'A'? Assume no flow in the pipe.

 h_A -3 x 0.9 + 0.375 x 0.9 - 0.375 x 13.6 = 0 h_A = 2.0625 m of water $p = \gamma x h$ = 9.81 x 21.99





p_{abs} = 101.3 +20.23 p_{abs} = 121.53 kPa

• DIFFERENTIAL MANOMETERS

Differential manometers are used to measure pressure difference between any two points. Common varieties of differential manometers are:

- (a) Two piezometers.
- (b) Inverted U-tube manometer.
- (c) U-tube differential manometers.
- (d) Micromanometers.

(a) Two Pizometers





The arrangement consists of two pizometers at the two points between which the pressure difference is required. The liquid will rise in both the piezometers. The difference in elevation of liquid levels can be recorded and the pressure difference can be calculated.

It has all the merits and demerits of piezometer.



(b) Inverted U-tube manometers

Inverted U-tube manometer is used to measure small difference in pressure between any two points. It consists of an inverted U-tube connecting the two points between which the pressure difference is required. In between there will be a lighter manometric liquid. Pressure difference between the two points can be calculated by writing the gauge equations for the system.

Let 'h_A' and 'h_B' be the pr head at 'A' and 'B' in meters of water $h_A - (Y_1 S_1) + (x S_M) + (y_2 S_2) = h_B.$ $h_A - h_B = S_1 y_1 - S_M x - S_2 y_2,$ $p_A - p_B = \gamma (h_A - h_B)$


(c) U-tube Differential manometers



A differential U-tube manometer is used to measure pressure difference between any two points. It consists of a U-tube containing heavier manometric liquid, the two limbs of which are connected to the gauge points between which the pressure difference is required. U-tube differential manometers can also be used for gases. By writing the gauge equation for the system pressure difference can be determined.

Let ' h_A ' and ' h_B ' be the pressure head of 'A' and 'B' in meters of water

$$\begin{split} h_{A} + S_{1} & Y_{1} + x \ S_{M} - Y_{2} \ S_{2} = h_{B} \\ h_{A} - h_{B} = Y_{2} \ S_{2} - Y_{1} \ S_{1} - x \ S_{M} \end{split}$$

Problems

(1) An inverted U-tube manometer is shown in figure. Determine the pressure difference between A and B in N/M^2 .

Let h_A and h_B be the pressure heads at A and B in meters of water.





$$\begin{split} h_A &- (190 \ x \ 10^{-2}) + (0.3 \ x \ 0.9) + (0.4) \ 0.9 = h_B \\ h_A &- h_B = 1.23 \ \text{meters of water} \\ p_A &- p_B = \gamma \left(h_A - h_B \right) = 9.81 \ x \ 1.23 \\ p_A &- p_B = 12.06 \ \text{kPa} \\ p_A &- p_B = 12.06 \ x \ 10^3 \ \text{N/m}^2 \end{split}$$

2. In the arrangements shown in figure. Determine the ho 'h'.



2.038 + 1.5 - (4 + 1.5 - h) 0.8 = -3.4

h = 3.6 m



 In the figure given, the air pressure in the left tank is 230 mm of Mercury (Vacuum). Determine the elevation of gauge liquid in the right limb at A. If the liquid in the right tank is water.





4. Compute the pressure different between 'M' and 'N' for the system shown in figure.



Let ' h_M ' and ' h_N ' be the pressure heads at M and N in m of water. hm + y x 1.15 - 0.2 x 0.92 + (0.3 - y + 0.2) 1.15 = hn hm + 1.15 y - 0.184 + 0.3 x 1.15 - 1.15 y + 0.2 x 1.15 = hn hm + 0.391 = hn hn - hm = 0.391 meters of water

$$p_n - p_m = \gamma (h_N - h_m)$$

= 9.81 x 0.391
$$p_n - p_m = 3.835 \text{ kPa}$$



5. Petrol of specify gravity 0.8 flows up through a vertical pipe. A and B are the two points in the pipe, B being 0.3 m higher than A. Connection are led from A and B to a U-tube containing Mercury. If the pressure difference between A and B is 18 kPa, find the reading of manometer.





$$\begin{aligned} h_A - h_B &= 1.835m \ of \ water \\ h_A + y \ x \ 0.8 - x \ 13.6 - (0.3 + y - x) \ 0.8 = h_B \\ h_A - h_B &= - \ 0.8y + \ 13.66 \ x + 0.24 + 0.8 \ y - 0.8 \ x \\ h_A - h_B &= 12.8 \ x + 0.24 \\ 1.835 &= 12.8 \ x + 0.24 \\ x &= 0.1246 \ m \end{aligned}$$



6. A cylindrical tank contains water to a height of 50mm. Inside is a small open cylindrical tank containing kerosene at a height specify gravity 0.8. The following pressures are known from indicated gauges.

 $p_B = 13.8 \text{ kPa} \text{ (gauge)}$ $p_C = 13.82 \text{ kPa} \text{ (gauge)}$

Determine the gauge pressure p_A and height h. Assume that kerosene is prevented from moving to the top of the tank.



```
\begin{split} h_{C} &= 1.409 \text{ m of water} \\ p_{B} &= 13.8 \text{ kPa} \\ h_{B} &= 1.407 \text{ meters of water} \\ 1.409 - 0.05 &= h_{A} \therefore h_{A} = 1.359 \text{ meters of water} \\ \therefore p_{A} &= 1.359 \text{ x } 9.81 \\ \therefore p_{A} &= 13.33 \text{ kPa} \\ h_{B} - h \text{ x } 0.8 - (0.05 - h) &= h_{A} \\ 1.407 - 0.8 \text{ h} - 0.05 + h &= 1.359 \\ 0.2 \text{ h} &= 1.359 - 1.407 + 0.05 \\ 0.2 \text{ h} &= 0.002 \\ h &= 0.02 \text{ m} \end{split}
```



7. What is the pressure p_A in the fig given below? Take specific gravity of oil as 0.8.



- $h_A + (3 \times 0.8) + (4.6 0.3) (13.6) = 0$ $h_A = 2.24 \text{ m of oil}$ $p_A = 9.81 \times 2.24$ $p_A = 21.97 \text{ kPa}$
- 8. Find 'd' in the system shown in fig. If $p_A = 2.7$ kPa





$$h_{A} = \frac{p_{A}}{\gamma} = \frac{2.7}{9.81}$$

$$h_{A} = 0.2752 m \text{ of water}$$

$$h_{A} + (0.05 x 0.6) + (0.05 + 0.02 - 0.01)0.6$$

$$+ (0.01 x 13.6) - (0.03 x 13.6) - d x 1.4) = 0$$

$$0.0692 - 1.4d = 0$$

$$d = 0.0494 m$$
or

 $d = 49.4 \, mm$

SINGLE COLUMN MANOMETER:

Single column manometer is used to measure small pressure intensities.



A single column manometer consists of a shallow reservoir having large cross sectional area when compared to cross sectional area of U – tube connected to it. For any



change in pressure, change in the level of manometeric liquid in the reservoir is small and change in level of manometric liquid in the U- tube is large.

To derive expression for pressure head at A:

BB and CC are the levels of manometric liquid in the reservoir and U-tube before connecting the point A to the manometer, writing gauge equation for the system we have,

+ y x S −
$$h_1$$
 x S_m = 0
∴ Sy = S_m h_1

Let the point A be connected to the manometer. B_1B_1 and C_1 C_1 are the levels of manometeric liquid. Volume of liquid between $BBB_1B_1 =$ Volume of liquid between CCC_1C_1

$$A\Delta = a h_2$$
$$\Delta = \frac{ah_2}{A}$$

Let ' h_A ' be the pressure head at A in m of water.

$$h_{A} + (y + \Delta) S - (\Delta + h_{1} + h_{2}) Sm = 0$$

$$h_{A} = (\Delta + h_{1} + h_{2}) Sm - (y + \Delta) S$$

$$= \Delta Sm + \underline{h_{1}}Sm + h_{2}Sm - \underline{yS} - \Delta S$$

$$h_{A} = \Delta (Sm - S) + h_{2}Sm$$

$$h_{A} = \frac{ah_{2}}{A} (Sm - S) + h_{2}Sm$$

:. It is enough if we take one reading to get 'h₂' If ' $\frac{a}{A}$ ' is made very small (by increasing 'A') then the I term on the RHS will be negligible.

Then $h_A = h_2 Sm$





INCLINED TUBE SINGLE COLUMN MANOMETER:

Inclined tube SCM is used to measure small intensity pressure. It consists of a large reservoir to which an inclined U – tube is connected as shown in fig. For small changes in pressure the reading ' h_2 ' in the inclined tube is more than that of SCM. Knowing the inclination of the tube the pressure intensity at the gauge point can be determined.

$$h_{A} = \frac{a}{A}h_{2}\sin\theta(Sm - S) + h_{2}\sin\theta.Sm$$

If ' $\frac{a}{A}$ ' is very small then $h_{A} = (h_{2} = \sin\theta)$ Sm

MECHANICAL GAUGES:

Pressure gauges are the devices used to measure pressure at a point. They are used to measure high intensity pressures where accuracy requirement is less.

Pressure gauges are separate for positive pressure measurement and negative pressure measurement. Negative pressure gauges are called Vacuum gauges.



BASIC PRINCIPLE:



Mechanical gauge consists of an elastic element which deflects under the action of applied pressure and this deflection will move a pointer on a graduated dial leading to the measurement of pressure. Most popular pressure gauge used is Bordon pressure gauge.

The arrangement consists of a pressure responsive element made up of phosphor bronze or special steel having elliptical cross section. The element is curved into a circular arc, one end of the tube is closed and free to move and the other end is connected to gauge point. The changes in pressure cause change in section leading to the movement. The movement is transferred to a needle using sector pinion mechanism. The needle moves over a graduated dial.



DIMENSIONAL ANALYSIS

It is a mathematical technique which makes use of study of dynamics as an art to the solution of engineering problems.

• Fundamental Dimensions

All physical quantities are measured by comparison which is made with respect to a fixed value.

Length, Mass and Time are three fixed dimensions which are of importance in fluid mechanics and fluid machinery. In compressible flow problems, temperature is also considered as a fundamental dimensions.

• Secondary Quantities or Derived Quantities

Secondary quantities are derived quantities or quantities which can be expressed in terms of two or more fundamental quantities.

• Dimensional Homogeneity

In an equation if each and every term or unit has same dimensions, then it is said to have Dimensional Homogeneity.

V = u + at
m/s m/s m/s² · s
$$LT^{-1} = (LT^{-1}) + (LT^{-2}) (T)$$

• Uses of Dimensional Analysis

- 1. It is used to test the dimensional homogeneity of any derived equation.
- 2. It is used to derive equation.
- 3. Dimensional analysis helps in planning model tests.

• Dimensions of quantities

1.	Length	LM ^o T ^o
2.	Mass	L ^o MT ^o
3.	Time	$L^{o}M^{o}T$
4.	Area	$L^2 M^0 T^0$



Velocity	$LM^{o}T^{-1}$
Acceleration	LM ^o T ⁻²
Momentum	LMT ⁻¹
Force	LMT ⁻²
Moment or Torque	$L^2 M T^{-2}$
Weight	LMT ⁻²
Mass density	$L^{-3}MT^{o}$
Weight density	$L^{-2}MT^{-2}$
Specific gravity	L ^o M ^o T ^o
Specific volume	$L^3M^{-1}T^o$
Volume flow rate	$L^3 M^{\circ} T^{-1}$
Mass flow rate	$L^{o}MT^{-1}$
Weight flow rate	LMT ⁻³
Work done	L^2MT^{-2}
Energy	L^2MT^{-2}
Power	$L^2 M T^{-3}$
Surface tension	L°MT ⁻²
Dynamic viscosity	$L^{-1}M^{+1}T^{-1}$
Kinematic viscosity	$L^2 M^0 T^{-1}$
Frequency	$L^{o}M^{o}T^{-1}$
Pressure	$L^{-1}MT^{-2}$
Stress	$L^{-1}MT^{-2}$
Е, С, К	$L^{-1}MT^{-2}$
Compressibility	$LM^{-1}T^2$
Efficiency	$L^{o}M^{o}T^{o}$
Angular velocity	$L^{o}M^{o}T^{-1}$
Thrust	LMT ⁻²
Energy head (Energy/unit mass)	$L^2 M^o T^{-2}$
Energy head (Energy/unit weight)	LM ^o T ^o
	 Velocity Acceleration Momentum Force Moment or Torque Weight Mass density Weight density Specific gravity Specific volume Volume flow rate Mass flow rate Weight flow rate Work done Energy Power Surface tension Dynamic viscosity Kinematic viscosity Frequency Pressure Stress E, C, K Compressibility Efficiency Angular velocity Energy/unit mass) Energy head (Energy/unit mass)



• Methods of Dimensional Analysis

There are two methods of dimensional analysis.

- 1. Rayleigh's method
- 2. Buckingham's (Π theorem) method

2. Rayleigh's method

Rayleigh's method of analysis is adopted when number of parameters or variables are less (3 or 4 or 5).

Methodology

 X_1 is a function of $X_2, X_3, X_4, \dots, X_n$ then it can be written as $X_1 = f(X_2, X_3, X_4, \dots, X_n)$ $X_1 = K (X_2^a, X_3^b, X_4^c, \dots)$

Taking dimensions for all the quantities

 $[X_1] = [X_2]^a [X_3]^b [X_4]^c \dots$

Dimensions for quantities on left hand side as well as on the right hand side are written and using the concept of Dimensional Homogeneity a, b, c can be determined.

Then,

 $X_1 = K \cdot X_2^{\ a} \cdot X_3^{\ b} \cdot X_4^{\ c} \cdot \dots$



- Problems 1: Velocity of sound in air varies as bulk modulus of elasticity K, Mass density ρ . Derive an expression for velocity in form $C = \sqrt{\frac{K}{\rho}}$
- Solution:

$$\begin{split} & C = f\left(K,\,\rho\right) \\ & C = M\cdot K^a\cdot\rho^b \end{split}$$

M – Constant of proportionality

$$[C] = [K]^{a} \cdot [\rho]^{b}$$

$$[LM^{o}T^{-1}] = [L^{-1}MT^{-2}]^{a} [L^{-3}MT^{o}]^{b}$$

$$[LM^{o}T^{-1}] = [L^{-a+(-36)}M^{a+b}T^{-2a}]$$

$$-a - 3b = 1$$

$$a + b = 0$$

$$-2a = -1$$

$$a = \frac{1}{2}$$

$$b = -\frac{1}{2}$$

$$C = MK^{1/2}\rho^{-1/2}$$

$$C = MK^{1/2}\rho^{-1/2}$$

$$If, M = 1, C = \sqrt{\frac{K}{\rho}}$$

$$\begin{array}{ll} C-Velocity & -LM^{o}T^{-1} \\ K-Bulk modulus & -L^{-1}MT^{-2} \\ \rho-Mass density & -L^{-3}MT^{o} \end{array}$$



- **Problem 2:** Find the equation for the power developed by a pump if it depends on head H discharge Q and specific weight γ of the fluid.
- Solution:

```
P = f (H, Q, \gamma)
P = K \cdot H^{a} \cdot Q^{b} \cdot \gamma^{c}
[P] = [H]^{a} \cdot [Q]^{b} \cdot [\gamma]^{c}
[L^{2}MT^{-3}] = [LM^{o}T^{o}]^{a} \cdot [L^{3}M^{0}T^{-1}]^{b} \cdot [L^{-2}MT^{-2}]^{c}
```

2 = a + 3b - 2c 1 = c -3 = -b - 2c -3 = -b - 2 b = -2 + 3 b = 1 2 = a + 3 - 2 a = 1 $P = K \cdot H^{1} \cdot Q^{1} \cdot \gamma^{1}$ $P = K \cdot H \cdot Q \cdot \gamma$ When, K = 1 $P = H \cdot Q \cdot \gamma$

Power	$= L^2 M T^{-3}$
Head	$= LM^{o}T^{o}$
Discharge	$= L^3 M^{\circ} T^{-1}$
Specific Weight	$= L^{-2}MT^{-2}$



• **Problem 3:** Find an expression for drag force R on a smooth sphere of diameter D moving with uniform velocity V in a fluid of density ρ and dynamic viscosity μ ..

• Solution:

```
\begin{split} &R = f\left(D, V, \rho, \mu\right) \\ &R = K \cdot D^{a} \cdot V^{b} \cdot \rho^{c}, \mu^{d} \\ &[R] = [D]^{a} \cdot [V]^{b} \cdot [\rho]^{c} \cdot [\mu]^{d} \\ &[LMT^{-2}] = [LM^{o}T^{o}]^{a} \cdot [LM^{o}T^{-1}]^{b} \cdot [L^{-3}MT^{o}]^{c} \cdot [L^{-1}MT^{-1}]^{d} \end{split}
```

c + d = 1 $c = 1 - d$
$-\mathbf{b} - \mathbf{d} = -2$ $\mathbf{b} = 2 - \mathbf{d}$
1 = a + b - 3c - d 1 = a + 2 - d - 3 (1 - d) - d 1 = a + 2 - d - 3 + 3d - d a = 2 - d
$R = K \cdot D^{2 \cdot d} \cdot V^{2 \cdot d} \cdot \rho^{1 \cdot d}, \mu^{d}$ $R = K \frac{D^{2}}{D^{d}} \cdot \frac{V^{2}}{V^{d}} \cdot \frac{\rho}{\rho^{d}} \cdot \mu^{d}$
$\mathbf{R} = \mathbf{K} \cdot \mathbf{\rho} \mathbf{V}^2 \mathbf{D}^2 \left[\frac{\mu}{\rho \mathbf{V} \mathbf{D}} \right]^d$
$\mathbf{R} = \rho \mathbf{V}^2 \mathbf{D}^2 \phi \left[\frac{\mu}{\rho \mathbf{V} \mathbf{D}} \right]$
$\mathbf{R} = \rho \mathbf{V}^2 \mathbf{D}^2 \phi \left[\frac{\rho \mathbf{V} \mathbf{D}}{\mu} \right]$
$\mathbf{R} = \boldsymbol{\rho} \mathbf{V}^2 \mathbf{D}^2 \boldsymbol{\phi} \left[\mathbf{N}_{\mathbf{R}\mathbf{e}} \right]$

Force	= LMT ⁻²	
Diameter	$= LM^{o}T^{o}$	
Velocity	$= LM^{o}T^{-1}$	
Mass density	$y = L^3 M T^{o}$	
Dynamic Viscosity = $L^{-1}MT^{-1}$		



•

Problem 4: The efficiency of a fan depends on the density ρ dynamic viscosity μ, angular velocity ω, diameter D, discharge Q. Express efficiency in terms of dimensionless parameters using Rayleigh's Method.

	$\eta - L^o M^o T^o$
Solution:	$\rho - L^{-3}MT^{o}$
$\eta = f(\rho, \mu, \omega, D, Q)$	μ - L ⁻¹ MT ⁻¹
$\eta = \mathbf{K} \cdot \boldsymbol{\rho}^{\mathrm{a}} \cdot \boldsymbol{\mu}^{\mathrm{b}} \cdot \boldsymbol{\omega}^{\mathrm{c}} \cdot \mathbf{D}^{\mathrm{d}} \cdot \mathbf{Q}^{\mathrm{e}}$	$\omega - L^{o}M^{o}T^{-1}$
$[\mathbf{n}] = [\mathbf{\rho}]^{a} \cdot [\mathbf{u}]^{b} \cdot [\mathbf{\omega}]^{c} \cdot [\mathbf{D}]^{d} \cdot [\mathbf{O}]^{e}$	D - LM ^o T ^o
	$O - L^3 M^0 T^{-1}$

 $[L^{o}M^{o}T^{o}] = [L^{-3}MT^{o}]^{a} \cdot [L^{-1}MT^{-1}]^{b} \cdot [L^{o}M^{o}T^{-1}]^{c} \cdot [LM^{o}T^{o}]^{d} \cdot [L^{3}M^{o}T^{-1}]^{e}$ $= [L^{-3a-b+d+3e}] [M^{a+b}] [T^{-b-c-e}]$

$$\mathbf{a} + \mathbf{b} = 0$$
$$\mathbf{a} = -\mathbf{b}$$

$$-b--e=0$$

$$\mathbf{c} = -\mathbf{b} - \mathbf{e}$$

$$-3a - b + d + 3e = o$$

+ $3b - b + d + 3e = 0$
 $d = -2b - 3e$

$$\therefore \eta = K \cdot \rho^{-b} \cdot \mu^{b} \cdot \omega^{-b-e} \cdot D^{-2b-3e} \cdot Q^{e}$$

$$\eta = K \cdot \frac{1}{\rho^{b}} \cdot \mu^{b} \frac{1}{\omega^{b} \cdot \omega^{e}} \cdot \frac{1}{(D^{2})^{b} \cdot (D^{3})^{e}} Q^{2}$$

$$\eta = K \left(\frac{\mu}{\rho \omega D^{2}}\right)^{b} \cdot \left(\frac{Q}{\omega D^{3}}\right)^{e}$$

$$\eta = \phi \left[\frac{\mu}{\rho \omega D^{2}}, \frac{Q}{\omega D^{3}}\right]$$



• **Problem 5:** The capillary rise H of a fluid in a tube depends on its specific weight γ and surface tension σ and radius of the tube R prove that $\frac{H}{R} = \phi \left[\frac{\sigma}{\gamma R^2} \right]$.

• Solution:

$$\begin{split} H &= f\left(\gamma, \sigma, R\right) \\ H &= K \cdot \gamma^{a} \cdot \sigma^{b} \cdot R^{c} \\ [H] &= \left[\gamma\right]^{a} \cdot \left[\sigma\right]^{b} \cdot \left[R\right]^{c} \\ [LM^{o}T^{o}] &= \left[L^{-2}MT^{-2}\right]^{a} \cdot \left[L^{o}MT^{-2}\right]^{b} \cdot \left[LM^{o}T^{o}\right]^{c} \\ [LM^{o}T^{o}] &= \cdot \left[L^{-2a+c} \cdot M^{a+b} \cdot T^{-2a-2b}\right] \\ -2a + c &= 1 \\ a + b &= 0 \\ -2a - 2b &= 0 \end{split} \qquad \begin{aligned} H - LM^{o}T^{o} \\ \gamma - L^{-2}MT^{-2} \\ \sigma - L^{o}MT^{-2} \\ R - LM^{o}T^{o} \end{aligned}$$

$$a = -b$$
$$c = 1 - 2b$$

$$H = K \cdot \gamma^{-b} \cdot \sigma^{b} \cdot R^{1-2b}$$
$$H = K \quad \frac{\sigma^{b}}{\gamma^{b}} \cdot \frac{R}{(R^{2})^{b}}$$
$$H = K \quad \left[\frac{\sigma}{\gamma R^{2}}\right]^{b}$$
$$\frac{H}{R} = \phi \left[\frac{\sigma}{\gamma R^{2}}\right].$$



3. Buckingham's IT Method

This method of analysis is used when number of variables are more.

Buckingham's Π Theorem

If there are n - variables in a physical phenomenon and those n-variables contain 'm' dimensions, then the variables can be arranged into (n-m) dimensionless groups called Π terms.

Explanation:

If f $(X_1, X_2, X_3, \dots, X_n) = 0$ and variables can be expressed using m dimensions then.

 $f(\Pi_1, \Pi_2, \Pi_3, \dots, \Pi_{n-m}) = 0$

Where, Π_1 , Π_2 , Π_3 , are dimensionless groups.

Each Π term contains (m + 1) variables out of which m are of repeating type and one is of non-repeating type.

Each Π term being dimensionless, the dimensional homogeneity can be used to get each Π term.

• Selecting Repeating Variables

- 1. Avoid taking the quantity required as the repeating variable.
- 2. Repeating variables put together should not form dimensionless group.
- 3. No two repeating variables should have same dimensions.
- 4. Repeating variables can be selected from each of the following properties.
 - a. Geometric property \rightarrow Length, height, width, area
 - b. Flow property \rightarrow Velocity, Acceleration, Discharge
 - c. Fluid property \rightarrow Mass density, Viscosity, Surface tension



Problem 1: Find an expression for drag force R on a smooth sphere of diameter D moving with uniform velocity V in a fluid of density ρ and dynamic viscosity μ .

Solution: •

 $f(R, D, V, \rho, \mu) = 0$ Here, n = 5, m = 3 \therefore Number of Π terms = (n - m) = 5 - 3 = 2 $\therefore f(\Pi_1, \Pi_2) = 0$

Let D, V, ρ be the repeating variables.

$$\Pi_1 = D^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot R$$

$$[L^{o}M^{o}T^{o}] = [L]^{a_{1}}[LT^{-1}]^{b_{1}}[ML^{-3}]^{c_{1}}[LMT^{-2}]$$
$$L^{o}M^{o}T^{o} = [L]^{a_{1+}b_{1-3}c_{1+1}}[M]^{c_{1+1}}[T]^{-b_{1-2}}$$

$$b_1 = 2$$

 $b_1 = -2$

$$c_{1} + 1 = 0$$

$$c_{1} = -1$$

$$a_{1} + b_{1} - 3_{c1} + 1 = 0$$

$$a_{1} + 2 + 3 + 1 = 0$$

$$a_{1} = -2$$

 $\Pi_1 = D^{-2} \cdot V^{-2} \cdot \rho^{-1} \cdot R$ R П

$$\Pi_1 = \frac{1}{\mathbf{D}^2 \mathbf{V}^2 \mathbf{p}}$$

$$\mathbf{D}^2 \mathbf{V}^2$$

$$\mathbf{D}^{a_2}$$
 \mathbf{v}^{b}

$$a_2$$
 b_2

$$D^{a_2} \cdot V^{b_2}$$

$$a_2 \cdot V^{U_2} \cdot \rho$$

$$\Pi_2 = D^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot \mu$$

 $[L^{o}M^{o}T^{o}] = [L]^{a_{2}}[LT^{-1}]^{b_{2}}[ML^{-3}]^{c_{2}}[L^{-1}MT^{-1}]$

 $[L^{o}M^{o}T^{o}] = [L]^{a_{2+}b_2-3c_2-1}[M]^{c_2+1}[T]^{-b_2-1}$

$$\frac{1}{\mathbf{D}^2 \mathbf{V}^2 \mathbf{o}}$$

$$c_1 = -1$$

$$a_1 + b_1 - 3_{c1} + 1 = 0$$

$$a_1 + b_1 = 3 + 1 = 0$$

$$a_1 + b_1 - 3_{c1} + 1 = 0$$

$$+b - 2 + 1 = 0$$

$$+ b_1 - 3 + 1 = 0$$

$$a_1 + b_1 - 3_{c1} + 1 = 0$$

$$-1 = -2$$

$$R = LMT^{-2}$$
$$D = LM^{o}T^{o}$$
$$V = LT^{-1}$$
$$\rho = ML^{-3}$$
$$\mu = L^{-1}MT^{-1}$$



$$-b_{2} - 1 = 0$$

$$b_{2} = -1$$

$$c_{2} = -1$$

$$a_{2} + b_{2} - 3c_{2} - 1 = 0$$

$$a_{2} - 1 + 3 - 1 = 0$$

$$a_{2} = -1$$

$$\Pi_{2} = D^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$$

$$\Pi_{2} = \frac{\mu}{\rho VD}$$

$$f(\Pi_{1}, \Pi_{2}) = 0$$

$$f\left(\frac{R}{D^{2}V^{2}\rho}, \frac{\mu}{\rho VD}\right) = 0$$

$$\frac{R}{D^{2}V^{2}\rho} = \phi\left(\frac{\mu}{\rho VD}\right)$$

$$R = \rho V^{2}D^{2}\phi\left(\frac{\mu}{\rho VD}\right)$$

- **Problem 2:** The efficiency of a fan depends on density ρ , dynamic viscosity μ , angular velocity ω , diameter D and discharge Q. Express efficiency in terms of dimensionless parameters.
- Solution:

	$f(\eta, \rho, \mu \omega, D, Q) = 0$	$\eta = L^{o}M^{o}T^{o}$
Here,	n = 6, m = 3	$\rho = ML^{-3}$
	\therefore Number of Π terms = 3	
	$\therefore f(\Pi_1, \Pi_2, \Pi_3) = 0$	$D = L$ $O = L^{3}T^{-1}$
		Q=L I

Г

Let, D, ω , ρ be the repeating variables.





$$[L^{\circ}M^{\circ}T^{\circ}] = [L]^{a_{1}}[T^{-1}]^{b_{1}}[ML^{-3}]^{c_{1}}[L^{-1}MT^{-1}]$$

$$[L^{\circ}M^{\circ}T^{\circ}] = [L]^{a_{1}-3c_{1}-1}[M]^{c_{1}+1}[T]^{-b_{1}-1}$$

$$b_{1} = -1$$

$$c_{1} = -1$$

$$a_{1} - 3c_{1} - 1 = 0$$

$$a_{1} = -2$$

$$\Pi_{1} = D^{-2} \cdot \omega^{-1} \cdot \rho^{-1} \cdot \mu$$

$$\Pi_{1} = \frac{\mu}{D^{2} \cdot \omega \cdot \rho}$$

$$\Pi_{2} = D^{a_{2}} \cdot \omega^{b_{2}} \cdot \rho^{c_{2}} \cdot Q$$

$$[L^{\circ}M^{\circ}T^{\circ}] = [L]^{a_{2}}[T^{-1}]^{b_{2}} [ML^{-3}]^{c_{2}}[L^{3}T^{-1}]$$

$$[L^{\circ}M^{\circ}T^{\circ}] = [L]^{a_{2}+3-3C2}[M]^{c_{2}}[T]^{-b_{2}-1}$$

$$c_{2} = 0$$

$$-b_{2} - 1 = 0$$

$$b_{2} = -1$$

$$a_{2} + 3 - 3c_{2} = 0$$

$$a_{2} + 3 = 0$$

$$a_{2} = -3$$

$$\Pi_{2} = D^{-3} \cdot \omega^{-1} \cdot \rho^{\circ} \cdot Q$$

$$\Pi_{2} = \frac{Q}{\omega D^{3}}$$

$$\Pi_{3} = D^{a_{3}} \cdot \omega^{b_{3}} \cdot \rho^{c_{3}} \cdot \eta$$

$$[L^{\circ}M^{\circ}T^{\circ}] = [L]^{a_{3}}[T^{-1}]^{b_{3}} [ML^{-3}]^{c_{3}}[M^{\circ}L^{\circ}T^{\circ}]$$

$$[L^{\circ}M^{\circ}T^{\circ}] = [L]^{a_{3}-3c_{3}}[M]^{c_{3}}[T]^{-b_{3}}$$

$$b_{3} = 0$$

95

 $\Pi_1 = D^{a_1} \cdot \omega^{b_1} \cdot \rho^{c_1} \cdot \mu$

$$c_{3} = 0$$

$$a_{3} = 0$$

$$\Pi_{3} = \eta$$

$$f(\Pi_{1}, \Pi_{2}, \Pi_{3}) = 0$$

$$f\left(\frac{\mu}{D^{2} \cdot \omega \cdot \rho}, \frac{\mu}{\omega D^{3}}, \eta\right) = 0$$

$$\eta = \phi\left(\frac{\mu}{D^{2}\omega\rho}, \frac{\mu}{\omega D^{3}}\right)$$

- Problem 3: The resisting force of a supersonic plane during flight can be considered as dependent on the length of the aircraft L, velocity V, viscosity μ, mass density ρ, Bulk modulus K. Express the fundamental relationship between resisting force and these variables.
- Solution:

f (R, L, K, μ, ρ, V) = 0
n = 6
∴ Number of Π terms =
$$6 - 3 = 3$$

∴ f (Π₁, Π₂, Π₃) = 0

Let, L, V, ρ be the repeating variables.

$$\Pi_{1} = L^{a_{1}} \cdot V^{b_{1}} \cdot \rho^{c_{1}} \cdot K$$

$$L^{o}M^{o}T^{o} = [L]^{a_{1}} [LT^{-1}]^{b_{1}} [ML^{-3}]^{c_{1}} [L^{-1}MT^{-2}]$$

$$L^{o}M^{o}T^{o} = [L]^{a_{1+}b_{1}-3c_{1}-1} [M]^{c_{1}+1} [T]^{-b_{1}-2}$$

$$b_{1} = -2$$

$$c_{1} = -1$$

$$a_{1} + b_{1} - 3c_{1} - 1 = 0$$

$$a_{1} - 2 + 3 - 1 = 0$$

$$a_{1} = 0$$



$$\begin{split} \Pi_{1} &= L^{\circ} \cdot V^{-1} \cdot \rho^{-1} \cdot K \\ \Pi_{1} &= \frac{K}{V^{2} \cdot \rho} \\ \Pi_{2} &= L^{a_{2}} \cdot V^{b_{2}} \cdot \rho^{c_{2}} \cdot R \\ L^{\circ}M^{\circ}T^{\circ} &= [L]^{a_{2}} [LT^{-1}]^{b_{2}} [ML^{-3}]^{c_{2}} [LMT^{-2}] \\ L^{\circ}M^{\circ}T^{\circ} &= [L]^{a_{2+}b_{2-}3c_{2}+1} [M]^{c_{2}+1} [T]^{-b_{2}-2} \\ -b_{2} - 2 &= 0 \\ b_{2} &= -2 \\ c_{2} &= -1 \\ \\ a_{2} + b_{2} - 3c_{2} + 1 &= 0 \\ a_{1} + 2 + 3 + 1 &= 0 \\ a_{2} &= -2 \\ \\ \Pi_{2} &= L^{-2} \cdot V^{-2} \cdot \rho^{-1} \cdot R \\ \Pi_{2} &= \frac{R}{L^{2}V^{2}\rho} \\ \\ \Pi_{3} &= L^{a_{3}} \cdot V^{b_{3}} \cdot \rho^{c_{3}} \cdot \mu \\ L^{\circ}M^{\circ}T^{\circ} &= [L]^{a_{3+}b_{3-}3c_{3}-1} [M]^{c_{3}+1} [T]^{-b_{3}-1} \\ -b_{3} - 1 &= 0 \\ c_{3} &= -1 \\ \end{split}$$



$$a_{3} + b_{3} - 3c_{3} - 1 = 0$$

$$a_{1} - 1 + 3 - 1 = 0$$

$$a_{3} = -1$$

$$\Pi_{3} = L^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$$

$$\Pi_{2} = \frac{\mu}{LV\rho}$$

$$f\left(\frac{K}{V^{2}\rho}, \frac{R}{L^{2}V^{2}\rho}, \frac{\mu}{LV\rho}\right) = 0$$

$$\frac{R}{L^{2}V^{2}\rho} = \phi\left(\frac{K}{V^{2}\rho}, \frac{\mu}{LV\rho}\right)$$

$$R = L^{2}V^{2}\rho \phi\left(\frac{K}{V^{2}\rho}, \frac{\mu}{LV\rho}\right)$$

- **Problem 4:** Using Buckingham Π theorem, show that velocity of fluid through a circular orifice is given by $V = \sqrt{2gH} \phi \left(\frac{D}{H}, \frac{\mu}{\rho VH}\right)$.
- Solution:

We have, f (V, D, H, μ , ρ , g) = 0 V = LT⁻¹, D = L, H = L, μ = L⁻¹MT⁻¹, ρ = ML⁻³, g = LT⁻² n = 6 m = 3 \therefore Number of Π terms = (6 - 3) = 3 Let H, g and ρ be repeating variables $\Pi_1 = H^{a_1} \cdot g^{b_1} \cdot \rho^{c_1} \cdot V$ [L^oM^oT^o] = [L]^{a_1}[LT⁻²]^{b_1}[ML⁻³]^{c_1}[LT⁻¹] [L^oM^oT^o] = [L]^{c_1+b_1-3c_1+1}[M]^{c_1}[T]^{-2b_1-1}



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$$\begin{aligned} -2b_{1} &= 1 \\ b_{1} &= -\frac{1}{2} \\ c_{1} &= 0 \end{aligned}$$

$$a_{1} + b_{1} - 3c_{1} + 1 &= 0 \\a_{1} &- \frac{1}{2} - 0 + 1 &= 0 \\a_{1} &= -\frac{1}{2} \end{aligned}$$

$$\Pi_{1} &= H^{-\frac{1}{2}} \cdot g^{-\frac{1}{2}} \cdot \rho^{o} \cdot V$$

$$\Pi_{1} &= \frac{V}{\sqrt{gH}}$$

$$\Pi_{2} &= H^{a_{2}} \cdot g^{b_{2}} \cdot \rho^{c_{2}} \cdot D$$

$$[L^{o}M^{o}T^{o}] &= [L]^{a_{2}} [LT^{-2}]^{b_{2}} [ML^{-3}]^{c_{2}} [L] \\[L^{o}M^{o}T^{o}] &= [L]^{a_{2,b}b_{2}-3c_{2}+1} [M]^{c_{2}} [T]^{-2b_{2}} \\-2b_{2} &= 0 \\b_{2} &= 0 \\c_{2} &= 0 \\a_{2} + b_{2} - 3c_{2} + 1 &= 0 \\a_{2} &= -1 \\\Pi_{2} &= H^{-1} \cdot g^{o} \cdot \rho^{o} \cdot D \\\Pi_{2} &= \frac{H}{D} \\\Pi_{3} &= H^{a_{3}} \cdot g^{b_{3}} \cdot \rho^{c_{3}} \cdot \mu \\[M^{o}L^{o}T^{o}] &= [L]^{a_{3}} [LT^{-2}]^{b_{3}} [ML^{-3}]^{c_{3}} [L^{-1}MT^{-1}] \end{aligned}$$



$$\begin{split} & [M^{o}L^{o}T^{o}] = [M]^{c_{3}+1} [L]^{a_{3+}b_{3-3}c_{3}-1} [T]^{-2b_{3}-1} \\ & c_{3} = -1 \\ & b_{3} = -\frac{1}{2} \\ & a_{3} - \frac{1}{2} + 3 - 1 = 0 \\ & c_{3} = -\frac{3}{2} \\ & \Pi_{3} = H^{-\frac{3}{2}} \cdot g^{-\frac{1}{2}} \cdot \rho^{-1} \cdot \mu \\ & \Pi_{3} = \frac{\mu}{\rho\sqrt{gH^{3}}} \\ & f(\Pi_{1}, \Pi_{2}, \Pi_{3}) = 0 \\ & f\left(\frac{V}{\sqrt{gH}}, \frac{H}{D}, \frac{\mu}{\rho\sqrt{gH + H}}\right) = 0 \\ & \frac{V}{\sqrt{gH}} = \phi\left(\frac{H}{D}, \frac{\mu}{\rho\sqrt{gH + H}}\right) \\ & v = \sqrt{2gH} \phi\left(\frac{D}{H}, \frac{\mu}{\rho V H}\right) \end{split}$$



- **Problem 5**: Using dimensional analysis, derive an expression for thrust P developed by a propeller assuming that it depends on angular velocity ω , speed of advance V, diameter D, dynamic viscosity μ , mass density ρ , elasticity of the fluid medium which can be denoted by speed of sound in the medium C.
- Solution:

$f(P, \omega, V, D, \mu, \rho, C) = 0$	$P = LMT^{-2}$
n = 7	$\omega = L^{o}M^{o}T^{-1}$
m = 3	$V = LM^{o}T^{-1}$
Taking D. V and o as repeating variables.	D = L
runing D, V and P as repeating variables.	$\mu = L^{-1}MT^{-1}$
$\Pi_1 = D^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot P$	$\rho = ML^{-3}$
	$C = LT^{-1}$

$$[L^{o}M^{o}T^{o}] = [L]^{a_{1}}[LT^{-1}]^{b_{1}}[ML^{-3}]^{c_{1}}[LMT^{-2}]$$
$$[L^{o}M^{o}T^{o}] = [L]^{c_{1+}b_{1}-3c_{1}+1}[M]^{c_{1}+1}[T]^{-b_{1}-2}$$

$$-b_{1}-2 = 0$$

$$b_{1} = -2$$

$$c_{1}+1 = 0$$

$$c_{1} = -1$$

$$a_{1}+b_{1}-3c_{1}+1 = 0$$

$$a_{1}-2+3+1 = 0$$

$$a_{1} = -2$$

 $\Pi_1 = \mathbf{D}^{-2} \cdot \mathbf{V}^{-2} \cdot \mathbf{\rho}^{-1} \cdot \mathbf{P}$

$$\Pi_1 = \frac{P}{D^2 V^2 \rho}$$

$$\Pi_{2} = D^{a_{2}} \cdot V^{b_{2}} \cdot \rho^{c_{2}} \cdot \omega$$
$$[L^{o}M^{o}T^{o}] = [L]^{a_{2}} [LT^{-2}]^{b_{2}} [ML^{-3}]^{c_{2}} [T^{-1}]$$



$$\Pi_{3} = \frac{\mu}{\rho V D}$$

$$\Pi_{4} = D^{a_{4}} \cdot V^{b_{4}} \cdot \rho^{c_{4}} \cdot C$$

$$M^{o}L^{o}T^{o} = [L]^{a_{4}} [LT^{-1}]^{b_{4}} [ML^{-3}]^{c_{4}} [LT^{-1}]$$

$$M^{o}L^{o}T^{o} = [L]^{a_{4}.b_{4}-3c_{4}+1} [M]^{c_{4}} [T]^{-b_{4}-1}$$

$$b_{4} = -1$$

$$c_{4} = 0$$

$$a_{4} - 1 + 0 + 1 = 0$$

$$a_{4} = 0$$

$$\begin{split} \left[M^{o}L^{o}T^{o}\right] &= \left[L\right]^{a_{3}}\left[LT^{-1}\right]^{b_{3}}\left[ML^{-3}\right]^{c_{3}}\left[L^{-1}MT^{-1}\right]\\ \left[M^{o}L^{o}T^{o}\right] &= \left[L\right]^{a_{3+}b_{3}-3c_{3}-1}\left[M\right]^{c_{3}+1}\left[T\right]^{-b_{3}-1}\\ b_{3} &= -1\\ c_{3} &= -1\\ c_{3} &= -1\\ n_{3} &= p^{-1}\cdot v^{-1}\cdot \rho^{-1}\cdot \mu \end{split}$$

 $\Pi_3 = D^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot \mu$

$$[L^{o}M^{o}T^{o}] = [L]^{a_{2+}b_{2}-3c_{2}} [M]^{c_{2}} [T]^{-b_{2}-1}$$

$$-b_{2} - 1 = 0$$

$$b_{2} = -1$$

$$c_{2} = 0$$

$$a_{2} - 1 + 0 = 0$$

$$a_{2} = -1$$

$$\Pi_{2} = D^{1} \cdot V^{-1} \cdot \rho^{o} \cdot \omega$$

$$\Pi_{2} = \frac{D\omega}{V}$$

$$\Pi_{4} = D^{o} \cdot V^{-1} \cdot \rho^{o} \cdot C$$

$$\Pi_{4} = \frac{C}{V}$$

$$f(\Pi_{1}, \Pi_{2}, \Pi_{3}, \Pi_{4}) = 0$$

$$f\left(\frac{P}{D^{2}V^{2}\rho}, \frac{D\omega}{V}, \frac{\mu}{\rho VD}, \frac{C}{V}\right) = 0$$

$$P = D^{2}V^{2}\rho \phi\left(\frac{D\omega}{V}, \frac{\mu}{\rho VD}, \frac{C}{V}\right)$$

• **Problem 6**: The pressure drop ΔP in a pipe depends on mean velocity of flow V, length of pipe l, viscosity of the fluid μ , diameter D, height of roughness projection K and mass density of the liquid ρ . Using Buckingham's method obtain an expression for ΔP .

• Solution:

$f(\Delta P, V, l, u, D, K, \rho) = 0$	$\Delta P = L^{-1}MT^{-2}$
n = 7	$V = LM^{o}T^{-1}$
\therefore number of Π terms = 7 - 3 = 4	$l = LM^{o}T^{o}$
	$\mu = L^{-1}MT^{-1}$
Let D, V, ρ be the repeating variables	$K = LM^{o}T^{o}$
$\Pi_1 = D^{a1} \cdot V^{b1} \cdot \rho^{c1} \cdot \Delta P$	$\rho = ML^{-3}$
$[L^{o}M^{o}T^{o}] = [L]^{a1} [LT^{-1}]^{b1} [ML^{-3}]^{c1} [L^{-1}MT^{-2}]$	
$[L^{o}M^{o}T^{o}] = [L]^{a1+b1-3c1-1}[M^{c1+1}][T^{-b1-2}]$	
$-b_1 - 2 = 0$	
$b_1 = -2$	
c ₁ = - 1	
$a_1 + b_1 - 3c_1 - 1 = 0$	
$a_1 - 2 + 3 - 1 = 0$	
$a_1 = 0$	





$$\begin{aligned} \Pi_{4} &= D^{a4} \cdot V^{b4} \cdot \rho^{c4} \cdot \mu \\ &[L^{o}M^{o}T^{o}] = [L]^{a4} [LT^{-1}]^{b4} [ML^{-3}]^{c4} [L^{-1}MT^{-1}] \\ &[L^{o}M^{o}T^{o}] = [L]^{a_{4+}b_{4}-3c_{4}-1} [M]^{c_{4}+1} [T]^{-b_{4}-1} \\ &b_{4} &= -1 \\ &c_{4} &= -1 \end{aligned}$$

$$\begin{split} \Pi_{3} &= D^{a3} \cdot V^{b3} \cdot \rho^{c3} \cdot K \\ &[L^{o}M^{o}T^{o}] = [L]^{a3} [LT^{-1}]^{b3} [ML^{-3}]^{c3} [L] \\ &[L^{o}M^{o}T^{o}] = [L]^{a3+b3-3c3+1} [M]^{c3} [T]^{-b3} \\ &b_{3} &= 0 \\ &c_{3} &= 0 \\ &a_{3} + b_{3} - 3c_{3} + 1 = 0 \\ &a_{3} &= -1 \\ &\Pi_{3} &= \frac{K}{D} \end{split}$$

$$\begin{split} \Pi_2 &= D^{a2} \cdot V^{b2} \cdot \rho^{c2} \cdot l \\ [L^o M^o T^o] &= [L]^{a2} [LT^{-1}]^{b2} [ML^{-3}]^{c2} [L] \\ [L^o M^o T^o] &= [L]^{a2+b2-3c2+1} [M]^{c2} [T]^{-b2} \\ b_2 &= 0 \\ c_2 &= 0 \\ a_2 &+ 0 + 0 + 1 = 0 \\ a_2 &= -1 \\ \end{split}$$

$$\begin{split} \Pi_2 &= D^{-1} \cdot V^o \cdot \rho^o \cdot L \\ \Pi_2 &= \frac{l}{D} \end{split}$$

$$\Pi_{1} = D^{o} \cdot V^{-2} \cdot \rho^{-1} \cdot \Delta P$$
$$\Pi_{1} = \frac{\Delta P}{V^{2} \cdot \rho}$$

$$a_{4} + b_{4} - 3c_{4} - 1 = 0$$

$$a_{4} - 1 + 3 - 1 = 0$$

$$a_{4} = -1$$

$$\Pi_{4} = D^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$$

$$\Pi_{4} = \frac{\mu}{\rho VD}$$

$$f\left(\frac{\Delta P}{V^{2}\rho}, \frac{l}{D}, \frac{K}{D}, \frac{\mu}{\rho VD}\right) = 0$$

$$\Delta P = \rho V^{2} \phi\left(\frac{L}{D}, \frac{K}{D}, \frac{\mu}{\rho VD}\right)$$



MODEL ANALYSIS

Before constructing or manufacturing hydraulics structures or hydraulics machines tests are performed on their models to obtain desired information about their performance. Models are small scale replica of actual structure or machine. The actual structure is called prototype.

• Similitude / Similarity

It is defined as the similarity between the prototype and it's model.

• Types of Similarity

There are three types of similarity.

- Geometric similarity
- Kinematic similarity
- Dynamic similarity

Geometrical Similarity

Geometric similarity is said to exist between the model and prototype if the ratio of corresponding linear dimensions between model and prototype are equal.

i.e.
$$\frac{L_{p}}{L_{m}} = \frac{h_{p}}{h_{m}} = \frac{H_{p}}{H_{m}} \dots = L_{r}$$

 $L_r \rightarrow$ scale ratio / linear ratio

$$\frac{A_p}{A_m} = (L_r)^2 \frac{V_p}{V_m} = (L_r)^3$$

• Kinematic Similarity

Kinematic similarity exists between prototype and model if quantities such at velocity and acceleration at corresponding points on model and prototype are same.

$$\frac{(V_1)_p}{(V_1)_m} = \frac{(V_2)_p}{(V_2)_m} = \frac{(V_3)_p}{(V_3)_m} \dots = V_r$$

 $V_r \rightarrow$ Velocity ratio

• Dynamic Similarity

Dynamic similarity is said to exist between model and prototype if ratio of forces at corresponding points of model and prototype is constant.



$$\frac{(F_1)_p}{(F_1)_m} = \frac{(F_2)_p}{(F_2)_m} = \frac{(F_3)_p}{(F_3)_m} \dots = F_R$$

 $F_R \rightarrow$ Force ratio

Dimensionless Numbers

Following dimensionless numbers are used in fluid mechanics.

- 1. Reynold's number
- 2. Froude's number
- 3. Euler's number
- 4. Weber's number
- 5. Mach number

1. Reynold's number

It is defined as the ratio of inertia force of the fluid to viscous force.

$$\therefore N_{\text{Re}} = \frac{F_{\text{i}}}{F_{\text{v}}}$$

Expression for N_{Re}

$$\begin{split} F_{i} &= Mass \; x \; Acceleration \\ F_{i} &= \rho \; x \; Volume \; x \; Acceleration \\ F_{i} &= \rho \; x \; Volume \; x \; \frac{Change \; in \; velocity}{Time} \\ F_{i} &= \rho \; x \; Q \; x \; V \\ F_{i} &= \rho \; A \; V^{2} \\ F_{V} &\Rightarrow \; Viscous \; force \\ F_{V} &= \tau \; x \; A \\ F_{V} &= \mu \; \frac{V}{y} \; A \\ F_{V} &= \mu \; \frac{V}{u} \; A \\ F_{V} &= \mu \; \frac{V}{L} \; A \\ N_{Re} &= \; \frac{\rho A \; V^{2}}{\mu \; \frac{V}{L} \; A} \end{split}$$



$$N_{Re} = \frac{\rho V L}{\mu}$$

In case of pipeline diameter is the linear dimension.

$$N_{Re} = \frac{\rho VD}{\mu}$$

2. Froude's Number (F_r)

It is defined as the ratio of square root of inertia force to gravity force.

$$F_r = \sqrt{\frac{F_i}{F_g}}$$

$$F_i = m \ x \ a$$

$$F_i = \rho \ x \ Volume \ x \ Acceleration$$

$$F_i = \rho AV^2$$

$$F_{g} = m x g$$

$$F_{g} = \rho x Volume x g$$

$$F_{g} = \rho x A x L x g$$

$$F_{\gamma} = \sqrt{\frac{\rho A V^{2}}{\rho x A x L x g}}$$
$$F_{\gamma} = \sqrt{\frac{V^{2}}{Lg}}$$
$$F_{\gamma} = \frac{V}{\sqrt{Lg}}$$


3. Euler's Number (ε_u)

It is defined as the square root of ratio of inertia force to pressure force.

$$\epsilon_{u} = \sqrt{\frac{F_{i}}{F_{p}}}$$

 F_i = Mass x Acceleration

$$F_{i} = \rho \ x \ Volume \ x \ \frac{Velocity}{Time}$$

$$F_{i} = \rho \ x \ Q \ x \ V$$

$$F_{i} = \rho A V^{2}$$

$$F_{p} = p \ x \ A$$

$$\varepsilon_{u} = \sqrt{\frac{\rho A V^{2}}{p A}} = V \sqrt{\frac{\rho}{p}}$$

$$\varepsilon_{u} = \frac{v}{\sqrt{\frac{\rho}{\rho}}}$$

4. Weber's Number (W_b)

It is defined as the square root of ratio of inertia force to surface tensile force.

$$W_{b} = \sqrt{\frac{F_{i}}{F_{p}}}$$

$$F_{b} = \rho A V^{2}$$

$$F_{s} = \sigma x L$$

$$W_{b} = \sqrt{\frac{\rho A V^{2}}{\sigma L}} = V \sqrt{\frac{\rho L}{\sigma}}$$

$$W_{b} = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$



5. Mach Number (M)

It is defined as the square root of ratio of inertia force to elastic force.

$$M = \sqrt{\frac{F_i}{F_e}}$$

$$F_i = \rho A V^2$$

$$F_e = K \times A$$

$$K \rightarrow Bulk \text{ modulus of elasticity}$$

$$A \rightarrow Area$$

$$M = \sqrt{\frac{\rho A V^2}{K A}}$$
$$M = \frac{V}{\sqrt{K / \rho}}$$
$$M = \frac{V}{C}$$

 $C \rightarrow$ Velocity of sound in fluid.



MODEL LAWS (SIMILARITY LAWS)

1. Reynold's Model Law

For the flows where in addition to inertia force, similarity of flow in model and predominant force, similarity of flow in model and prototype can be established if Re is same for both the system.

This is known as Reynold's Model Law.

Re for model = Re for prototype

$$(N_{Re})_{m} = (N_{Re})_{p}$$
$$\left(\frac{\rho VD}{\mu}\right)_{m} = \left(\frac{\rho VD}{\mu}\right)_{p}$$
$$\frac{\rho_{m} \cdot V_{m} \cdot D_{m}}{\rho_{p} \cdot V_{p} \cdot D_{p}} = 1$$
$$\frac{\mu_{m}}{\mu_{p}}$$
$$\frac{\rho_{r} \cdot V_{r} \cdot D_{r}}{\mu_{r}} = 1$$

Applications:

- i) In flow of incompressible fluids in closed pipes.
- ii) Motion of submarine completely under water.
- iii) Motion of air-planes.

2. Froude's Model Law

When the force of gravity is predominant in addition to inertia force then similarity can be established by Froude's number. This is known as Froude's model law.

$$(F_{r})_{m} = (F_{r})_{p}$$
$$\left(\frac{V}{\sqrt{gL}}\right)_{m} = \left(\frac{V}{\sqrt{gL}}\right)$$
$$\left(\frac{V}{\sqrt{gL}}\right)_{r} = 1$$



Applications:

- i) Flow over spillways.
- ii) Channels, rivers (free surface flows).
- iii) Waves on surface.
- iv) Flow of different density fluids one above the other.

3. Euler's Model Law

When pressure force is predominant in addition to inertia force, similarity can be established by equating Euler number of model and prototype. This is called Euler's model law.

$$(\varepsilon_{u})_{m} = (\varepsilon_{u})_{p}$$

$$\left(\frac{V_{m}}{\sqrt{p_{m}/p_{m}}}\right) = \left(\frac{V_{p}}{\sqrt{p_{p}/p_{p}}}\right)$$

Application:

Turbulent flow in pipeline where viscous force and surface tensile forces are entirely absent.

4. Mach Model Law

In places where elastic forces are significant in addition to inertia, similarity can be achieved by equating Mach numbers for both the system.

This is known as Mach model law.

$$\mathbf{M}_{m} = \mathbf{M}_{\gamma}$$

$$\left(\frac{\mathbf{V}_{m}}{\sqrt{\mathbf{K}_{m}/\mathbf{\rho}_{m}}}\right) = \left(\frac{\mathbf{V}_{p}}{\sqrt{\mathbf{K}_{p}/\mathbf{\rho}_{p}}}\right)$$



$$\left(\frac{V_{\gamma}}{\sqrt{K_{\gamma}/\rho_{\gamma}}}\right) = 1$$

Applications:

- i) Aerodynamic testing where velocity exceeds speed of sound.Eg: Flow of airplane at supersonic speed.
- ii) Water hammer problems.

5. Weber's Model Law:

If surface tension forces are predominant with inertia force, similarity can be established by equating Weber number of model and prototype.

$$W_{m} = W_{\gamma}$$

$$\left(\frac{V}{\sqrt{\sigma/\rho L}}\right)_{m} = \left(\frac{V}{\sqrt{\sigma/\rho L}}\right)_{p}$$

$$\left(\frac{V}{\sqrt{\omega/\rho L}}\right)_{r} = 1$$

Applications:

- i) Flow over wires with low heads.
- ii) Flow of very thin sheet of liquid over a surface.
- iii) Capillary flows.



- **Problem 1**: A pipe of diameter 1.5 m is required to transmit an oil of S = 0.9 and viscosity 3 x 10^{-2} poise at 3000 lps. Tests were conducted on 15 cm diameter pipe using water at 20°C. Find velocity and rate of flow of model if μ water at 20°C is 0.01 poise.
- Solution

$$\begin{array}{l} D_p = 1.5 \ m\\ S_p = 0.9\\ \mu_p = 3 \ x \ 10^{-2} \ poise = 3 \ x \ 10^{-3} \ Ns/m^2\\ Q_p = 3000 \ lps = 3000 \ x \ 10^{-3} \ m^3/s = 3 \ m^3/s\\ D_m = 0.15 \ m\\ S_m = 1\\ V_m = ?\\ Q_m = ?\\ A_p \ V_p = Q_p\\ V_p = 1.698 \ m/s\\ \mu_m = 0.01 \ poise\\ = 0.001 \ poise\\ \rho_m = 1000 \ kg/m^3\\ \rho_p = 0.9 \ x \ 1000 = 900 \ kg/m^3 \end{array}$$

$$(R_{e})_{m} = (R_{e})_{p}$$

$$\frac{\rho_{m} V_{m} D_{m}}{\mu_{m}} = \frac{\rho_{p} V_{p} D_{p}}{\mu_{p}}$$

$$\frac{1000 \times V_{m} \times 0.15}{0.001} = \frac{900 \times 1.698 \times 1.5}{3 \times 10^{-3}}$$

$$V_{m} = 5.094 \text{ m/s}$$

$$Q = A_{m}V_{m}$$

$$Q = A_{m}V_{m}$$

$$Q = \frac{\Pi}{4} (0.15)^{2} (5.094)$$

$$Q = 0.09 \text{ m}^{3}/\text{s}$$

$$Q = 90 \text{ lps.}$$



- **Problem 2**: In a 1 in 40 model of spillway velocity and discharge are 2 m/s and 2.5 m³/s. Find the corresponding velocity and discharge in prototype.
- Solution

$$L_{\gamma} = \frac{L_m}{L_p} = \frac{1}{40}$$
$$V_m = 2 \text{ m/s}$$
$$Q_m = 2.5 \text{ m}^3/\text{s}$$

Since it is a spillway problem, Froude's law of similarity is used.

$$(F_{\gamma})_{m} = (F_{\gamma})_{p}$$
$$\left(\frac{V}{\sqrt{gL}}\right)_{m} = \left(\frac{V}{\sqrt{gL}}\right)_{p}$$
$$\frac{2}{\sqrt{9.81 \text{ x } 1}} = \frac{V_{p}}{\sqrt{9.81 \text{ x } 40}}$$
$$V_{p} = 12.65 \text{ m/s}$$

For a spillway,

$$Q\alpha L^{2.5}$$

$$\frac{Q_{p}}{Q_{m}} = \frac{L_{p}^{2.5}}{L_{m}^{2.5}}$$

$$\frac{Q_{p}}{2.5} = (40)^{2.5}$$

$$Q_{p} = 25298.22 \text{ m}^{3}/\text{s}$$

• **Problem 3**: Experiments area to be conducted on a model ball which is twice as large as actual golf ball. For dynamic similarity, find ratio of initial velocity of model to that of actual ball. Take fluid in both cases as air at STP.

It is a case of motion of fully submerged body.

: Reynolds's number of flow determines dynamic similarity.



• Solution

$$\therefore (\mathbf{R}_{e})_{m} = (\mathbf{R}_{e})_{p}$$

$$\rho_{m} = \rho_{p}$$

$$\mu_{m} = \mu_{p}$$

$$\left(\frac{\rho V d}{\mu}\right)_{m} = \left(\frac{\rho V d}{\mu}\right)_{p}$$

$$\frac{d_{m}}{d_{p}} = 2$$

$$V_{m} \cdot d_{m} = V_{p} \cdot d_{p}$$

$$\frac{V_{m}}{V_{p}} = \frac{d_{p}}{d_{m}}$$

$$\frac{V_{m}}{V_{p}} = \frac{1}{2}$$

$$V_{m} = 0.5 V_{p}$$

• **Problem 4**: Water at 15°C flows at 4 m/s in a 150 mm diameter pipe. At what velocity oil at 30°C must flow in a 75 mm diameter pipe for the flows to be dynamically similar? Take kinematic viscosity of water at 15°C as $1.145 \times 10^{-6} \text{ m}^2/\text{s}$ and that for oil at 30°C as $3 \times 10^{-6} \text{ m}^2/\text{s}$.

• Solution

$$V_{p} = 4 \text{ m/s}$$

$$d_{p} = 0.15 \text{ m}$$

$$V_{m} = ?$$

$$D_{m} = 0.075 \text{ m}$$

$$\left(\frac{\mu}{\rho}\right)_{p} = 1.145 \text{ x } 10^{-6} \text{ m}^{2}/\text{s}$$

$$\left(\frac{\mu}{\rho}\right)_{m} = 3 \text{ x } 10^{-6} \text{ m}^{2}/\text{s}$$

$$\left(\frac{\rho \text{Vd}}{\mu}\right)_{m} = \left(\frac{\rho \text{Vd}}{\mu}\right)_{p}$$



$$\frac{V_{m} \ge 0.075}{3 \ge 10^{-6}} = \frac{4 \ge 0.15}{1.145 \ge 10^{-6}}$$
$$V_{m} = 20.96 \text{ m/s}$$

- **Problem 5**: A model with linear scale ratio (model to prototype) x, of a mach 2 supersonic aircraft is tested in a wind tunnel where in pressure is y times the atmospheric pressure. Determine the speed of model in tunnel given that velocity of sound m atmospheric air is Z.
- Solution:

$$\frac{L_{m}}{L_{p}} = x$$

$$M = 2$$

$$P_{m} = y p_{atm}$$

$$\rho_{m} = y \rho_{atm}$$

$$C = Z$$

$$\frac{V}{C} = 2$$

$$\frac{V}{Z} = 2$$

$$V_{p} = 2Z$$

Dynamic similarity in this case is established by Reynold's Model law.

$$(R_{e})_{m} = (R_{e})_{p}$$

$$\left(\frac{\rho VL}{\mu}\right)_{m} = \left(\frac{\rho VL}{\mu}\right)_{p}$$

$$y \frac{\rho_{atm} \times V_{m} \times L_{m}}{\mu_{m}} = \frac{\rho_{p} \times V_{p} \times L_{p}}{\mu_{p}}$$

$$y \frac{\rho_{atm} \cdot V_{m} \cdot x}{\mu_{m}} = \rho_{atm} \cdot 2Z$$

$$V_{m} = \frac{2Z}{xy}$$

