SIGNAL FLOW GRAPHS

The relationship between an input variable and an output variable of a signal flow graph is given by the net gain between input and output nodes and is known as overall gain of the system. Masons gain formula is used to obtain the over all gain (transfer function) of signal flow graphs.

Masons Gain Formula

Gain P is given by

$$P = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

Where, P_k is gain of k^{th} forward path,
Δ is determinant of graph

Δ=1-(sum of all individual loop gains)+(sum of gain products of all possible combinations of two nontouching loops – sum of gain products of all possible combination of three nontouching loops) + ···

Δ_k is cofactor of k^{th} forward path determinant of graph with loops touching k^{th} forward path. It is obtained from Δ by removing the loops touching the path P_k.

Example 1
Obtain the transfer function of C/R of the system whose signal flow graph is shown in Fig.1

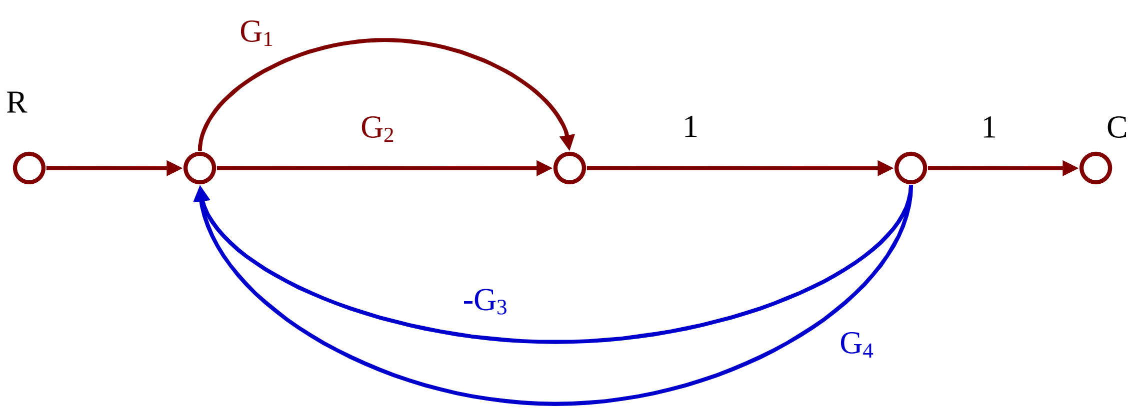


Figure 1 Signal flow graph of example 1

There are two forward paths:
Gain of path 1 : $P_1=G_1$
Gain of path 2 : $P_2=G_2$

There are four loops with loop gains:
$L_1=-G_1G_3$, $L_2=G_1G_4$, $L_3= -G_2G_3$, $L_4= G_2G_4$
There are no non-touching loops.
$\Delta = 1+G_1G_3-G_1G_4+G_2G_3-G_2G_4$
Forward paths 1 and 2 touch all the loops. Therefore, $\Delta_1= 1$, $\Delta_2= 1$

The transfer function T = $\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 + G_2}{1 + G_1G_3 - G_1G_4 + G_2G_3 - G_2G_4}$

Example 2
Obtain the transfer function of C(s)/R(s) of the system whose signal flow graph is shown in Fig.2.

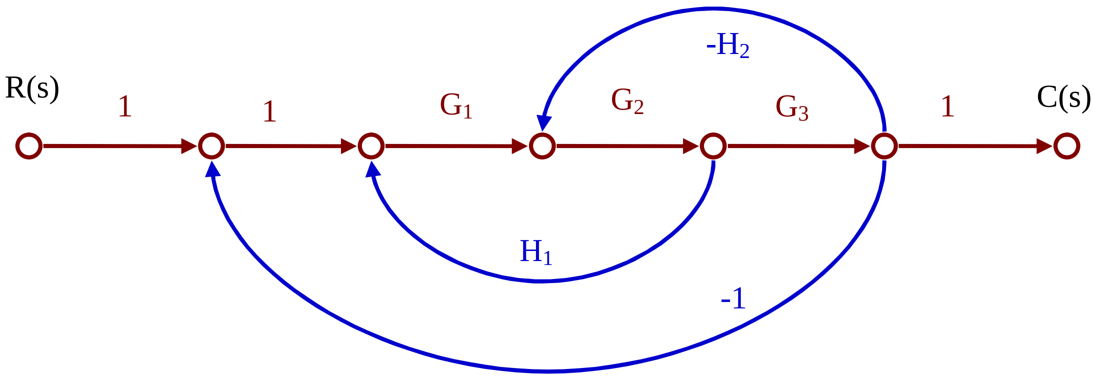


Figure 2 Signal flow graph of example 2

There is one forward path, whose gain is: $P_1=G_1G_2G_3$
There are three loops with loop gains:
$L_1=-G_1G_2H_1$, $L_2=G_2G_3H_2$, $L_3= -G_1G_2G_3$
There are no non-touching loops.
$\Delta = 1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3$
Forward path 1 touches all the loops. Therefore, $\Delta_1= 1$.
The transfer function T = $\frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_1G_3H_2 + G_1G_2G_3}$

Example 3

Obtain the transfer function of C(s)/R(s) of the system whose signal flow graph is shown in Fig.3.

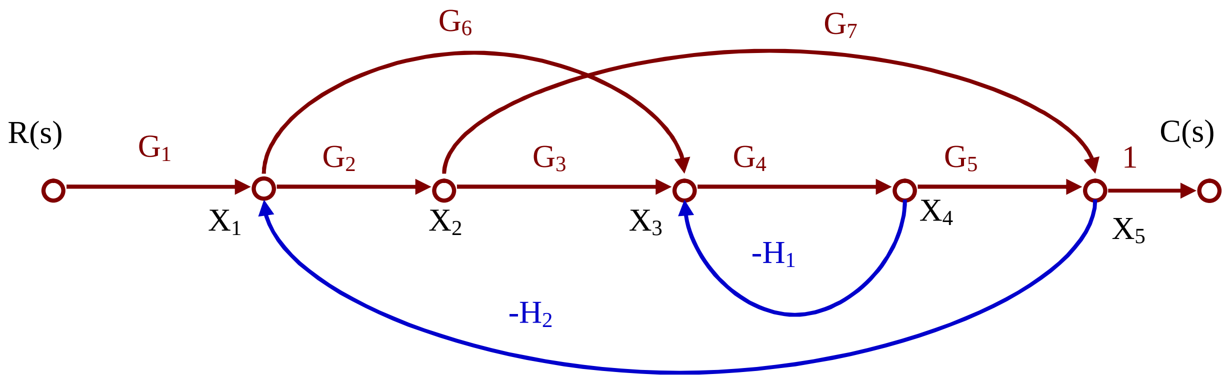


Figure 3 Signal flow graph of example 3

There are three forward paths.

The gain of the forward path are: $P_1=G_1G_2G_3G_4G_5$

$P_2=G_1G_6G_4G_5$

$P_3= G_1G_2G_7$

There are four loops with loop gains:

$L_1=-G_4H_1$, $L_2=-G_2G_7H_2$, $L_3= -G_6G_4G_5H_2$, $L_4=-G_2G_3G_4G_5H_2$

There is one combination of Loops L_1 and L_2 which are nontouching with loop gain product $L_1L_2=G_2G_7H_2G_4H_1$

$\Delta = 1+G_4H_1+G_2G_7H_2+G_6G_4G_5H_2+G_2G_3G_4G_5H_2+ G_2G_7H_2G_4H_1$

Forward path 1 and 2 touch all the four loops. Therefore $\Delta_1= 1$, $\Delta_2= 1$.

Forward path 3 is not in touch with loop1. Hence, $\Delta_3= 1+G_4H_1$.

The transfer function T =

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta} = \frac{G_1G_2G_3G_4G_5 + G_1G_4G_5G_6 + G_1G_2G_7(1+G_4H_1)}{1+G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_2G_4G_7H_1H_2}$$

Example 4

Find the gains $\frac{X_6}{X_1}, \frac{X_5}{X_2}, \frac{X_3}{X_1}$ for the signal flow graph shown in Fig.4.

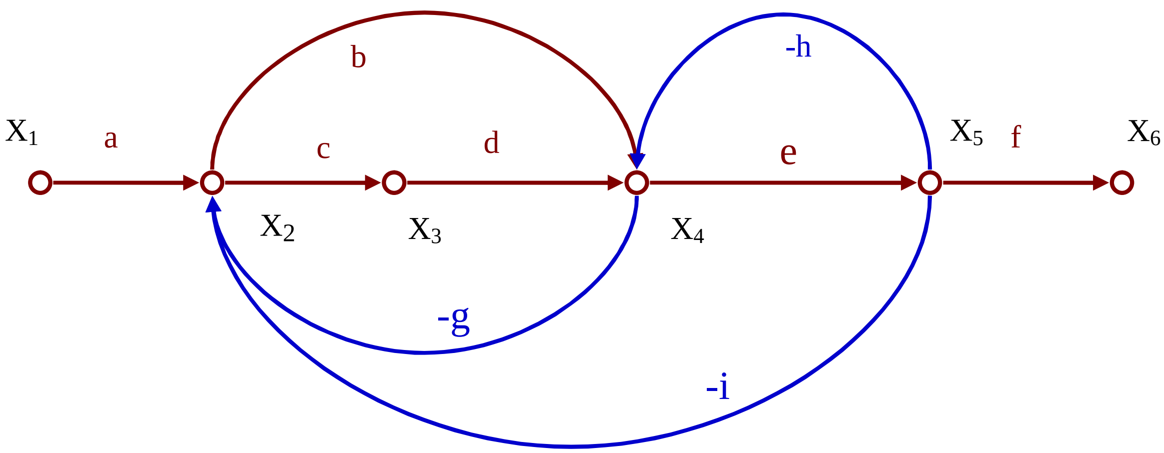


Figure 4 Signal flow graph of MIMO system

Case 1: $\frac{X_6}{X_1}$

There are two forward paths.
The gain of the forward path are: P_1=acdef
P_2=abef
There are four loops with loop gains:
L_1=-cg, L_2=-eh, L_3= -cdei, L_4=-bei
There is one combination of Loops L_1 and L_2 which are nontouching with loop gain product L_1L_2=cgeh
Δ = 1+cg+eh+cdei+bei+cgeh
Forward path 1 and 2 touch all the four loops. Therefore Δ_1= 1, Δ_2= 1.

The transfer function T = $\frac{X_6}{X_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{cdef + abef}{1 + cg + eh + cdei + bei + cgeh}$

Case 2: $\frac{X_5}{X_2}$

The modified signal flow graph for case 2 is shown in Fig.5.

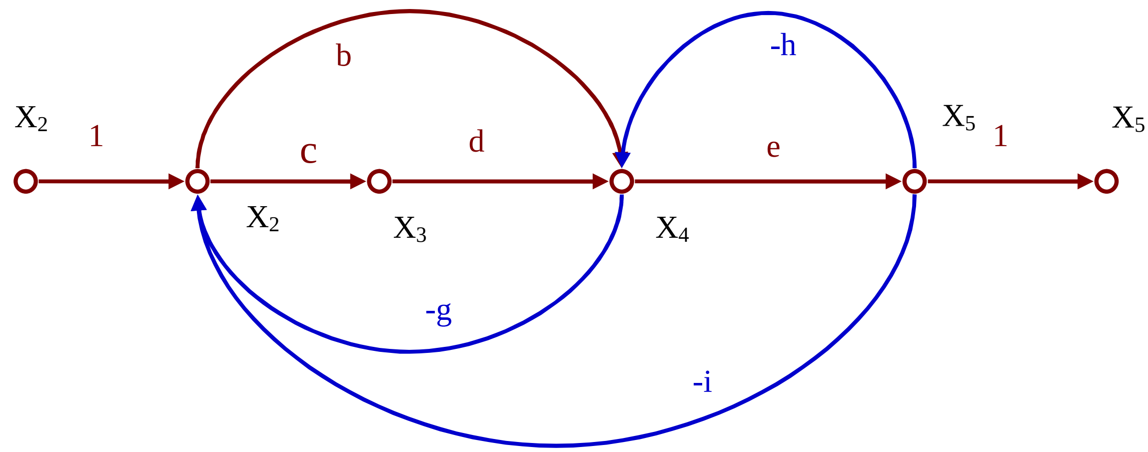


Figure 5 Signal flow graph of example 4 case 2

The transfer function can directly manipulated from case 1 as branches a and f are removed which do not form the loops. Hence,

The transfer function T= $\frac{X_5}{X_2} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{cde + be}{1 + cg + eh + cdei + bei + cgeh}$

Case 3: $\frac{X_3}{X_1}$

The signal flow graph is redrawn to obtain the clarity of the functional relation as shown in Fig.6.

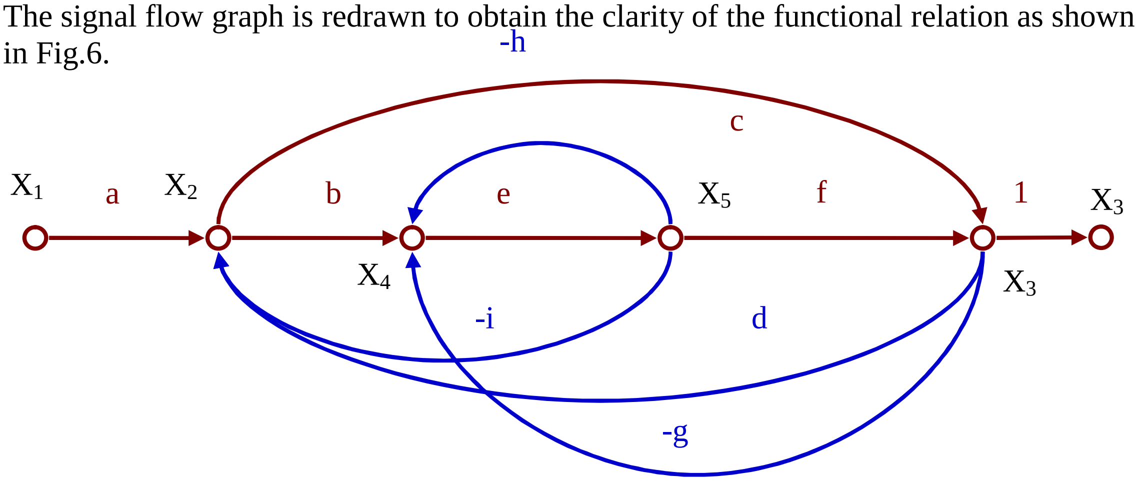


Figure 6 Signal flow graph of example 4 case 3

There are two forward paths.
The gain of the forward path are: P_1=abcd
P_2=ac
There are five loops with loop gains:
L_1=-eh, L_2=-cg, L_3= -bei, L_4=edf, L_5=-befg
There is one combination of Loops L_1 and L_2 which are nontouching with loop gain product L_1L_2=ehcg
Δ = 1+eh+cg+bei+efd+befg+ehcg
Forward path 1 touches all the five loops. Therefore Δ_1= 1.
Forward path 2 does not touch loop L_1. Hence, Δ_2= 1+ eh

The transfer function T = $\frac{X_3}{X_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{abef + ac(1+eh)}{1+eh+cg+bei+efd+befg+ehcg}$

Example 5

For the system represented by the following equations find the transfer function X(s)/U(s) using signal flow graph technique.

$$X = X_1 + \ {}_3u$$

$$\dot{X}_1 = -a_1X_1 + X_2 + \ {}_2u$$

$$\dot{X}_2 = -a_2X_1 + \ {}_1u$$

Taking Laplace transform with zero initial conditions

$$X(s) = X_1(s) + \ {}_3U(s)$$
$$sX_1(s) = -a_1X_1(s) + X_2(s) + \ {}_2U(s)$$
$$sX_2(s) = -a_2X_1(s) + \ {}_1U(s)$$

Rearrange the above equation

$$X(s) = X_1(s) + \ {}_3U(s)$$

$$X_1(s) = \frac{-a_1}{s}X_1(s) + \frac{1}{s}X_2(s) + \frac{{}_2}{s}U(s)$$

$$X_2(s) = \frac{-a_2}{s}X_1(s) + \frac{{}_1}{s}U(s)$$

The signal flow graph is shown in Fig.7.

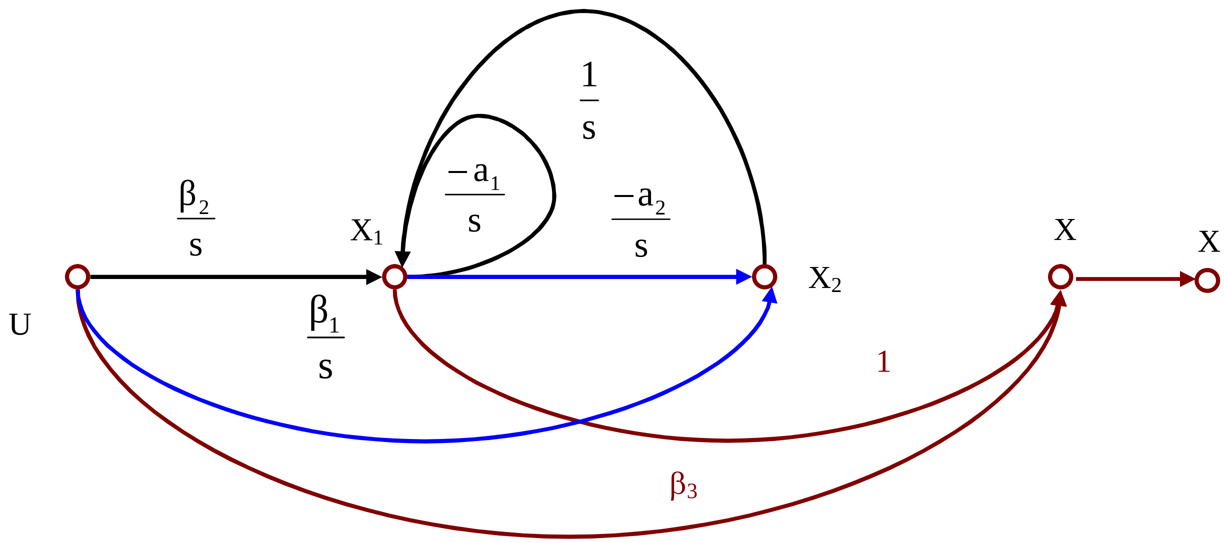


Figure 7 Signal flow grapgh of example 5

There are three forward paths.
The gain of the forward path are: $P_1=\beta_3$
$P_2=\beta_1/s^2$
$P_3=\beta_2/s$

There are two loops with loop gains:

$$L_1=\frac{-a_1}{s}$$

$$L_2=\frac{-a_2}{s^2}$$

L_1=-eh, L_2=-cg, L_3= -bei, L_4=edf, L_5=-befg

There are no combination two Loops which are nontouching.

$$\Delta=1+\frac{a_1}{s}+\frac{a_2}{s^2}$$

Forward path 1 does not touch loops L_1 and L_2. Therefore

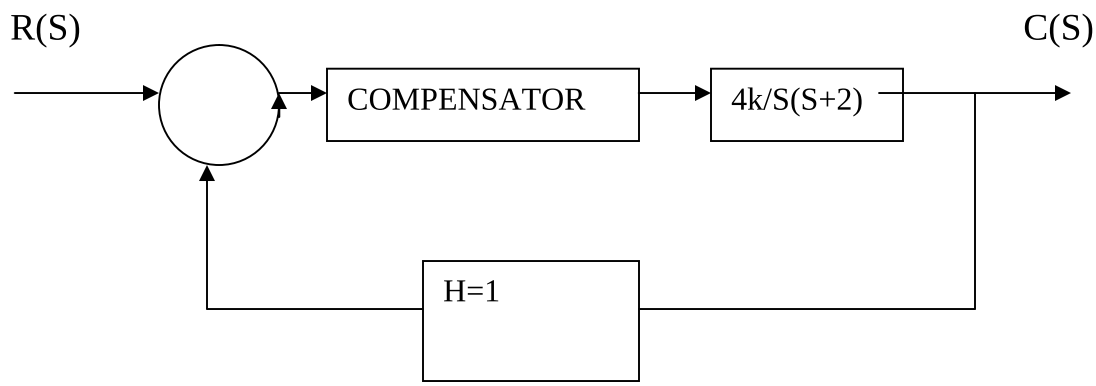
$$\Delta_1=1+\frac{a_1}{s}+\frac{a_2}{s^2}$$

Forward path 2 path 3 touch the two loops. Hence, Δ_2= 1, Δ_2= 1.

The transfer function T = $\frac{X_3}{X_1}=\frac{P_1\Delta_1+P_2\Delta_2+P_3\Delta_3}{\Delta}=\frac{{}_3\left(s^2+a_1s+a_2\right)+{}_2s+{}_1}{s^2+a_1s+a_2}$

SYSTEM COMPENSATION

PROBLEM - Design a suitable compensator for the system whose OLTF is as shown in the block diagram. Static velocity error coefficient is 20 sec-1the phase margin is at least 50 and gain margin is at least 10 db



STEP1

- From the performance specifications determine the value of K from Kv
- Kv= Lim (s-0) S. G(S) =20

1

- From this Lim (s-0) s 4K/(s(s+2)) = 20
- K=10 as s-0

STEP2

Obtain the BODE PLOT for the same

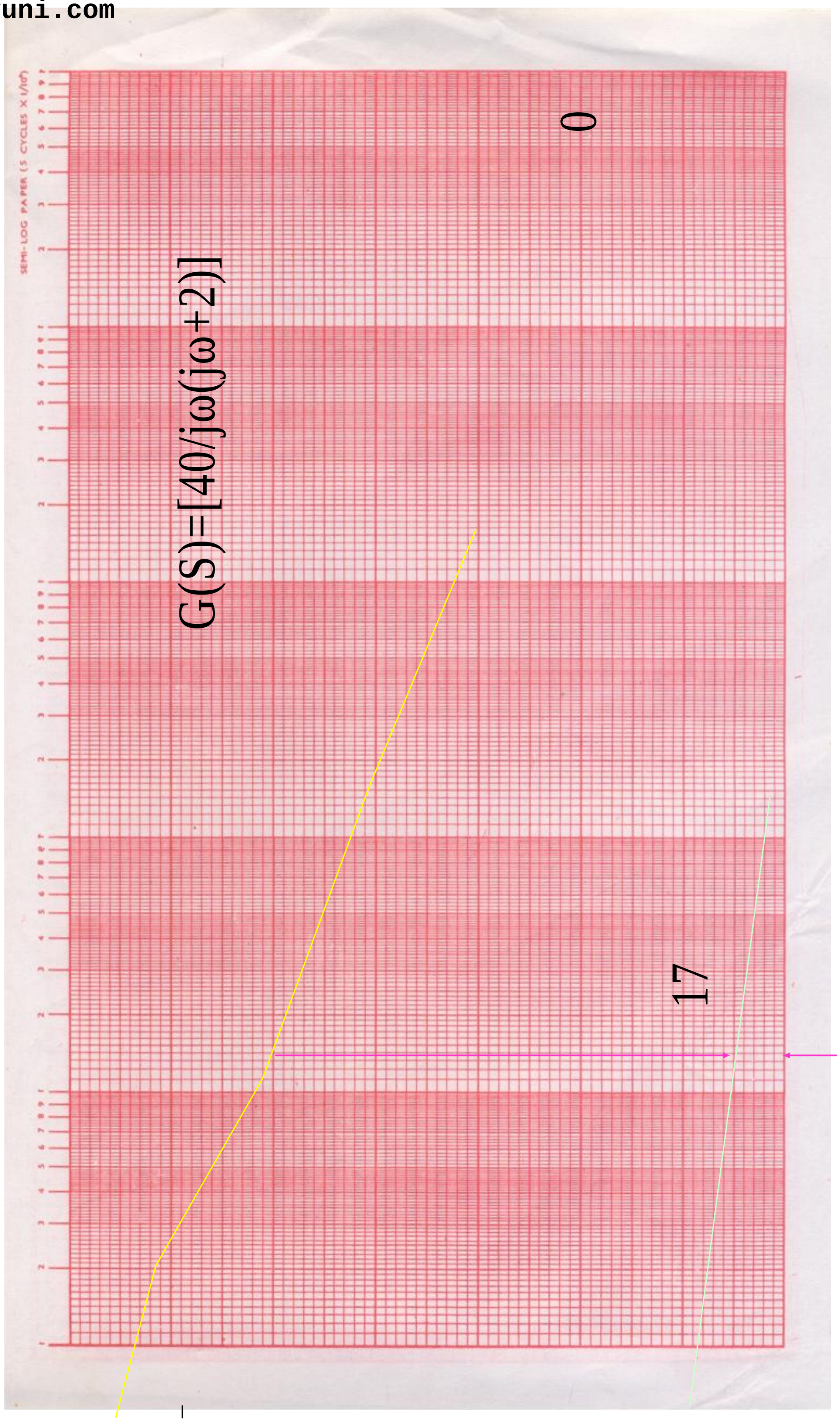
Obtain the Gain Margin and Phase Margin for the obtained value of K=10

From the plot Phase margin is 17

Gain margin is +∞ db

Additional Phase lead necessary is +33

In order to compensate for the shift in the gain crossover frequency +5 degree added


G(S)=[40/jω(jω+2)]
0
17

STEP3

For a value of +38 degrees obtain ά from

Sin Φm = { (1- ά)/ (1+ ά)}

Attenuation factor ά is 0.24

Maximum Phase Lead Angle Φm occurs at the geometric mean of two corner frequencies

Obtained from

Log ωm = ½ [log (1/T) = log(1/ ά T)]

ωm = [1/√ ά T]

STEP 4

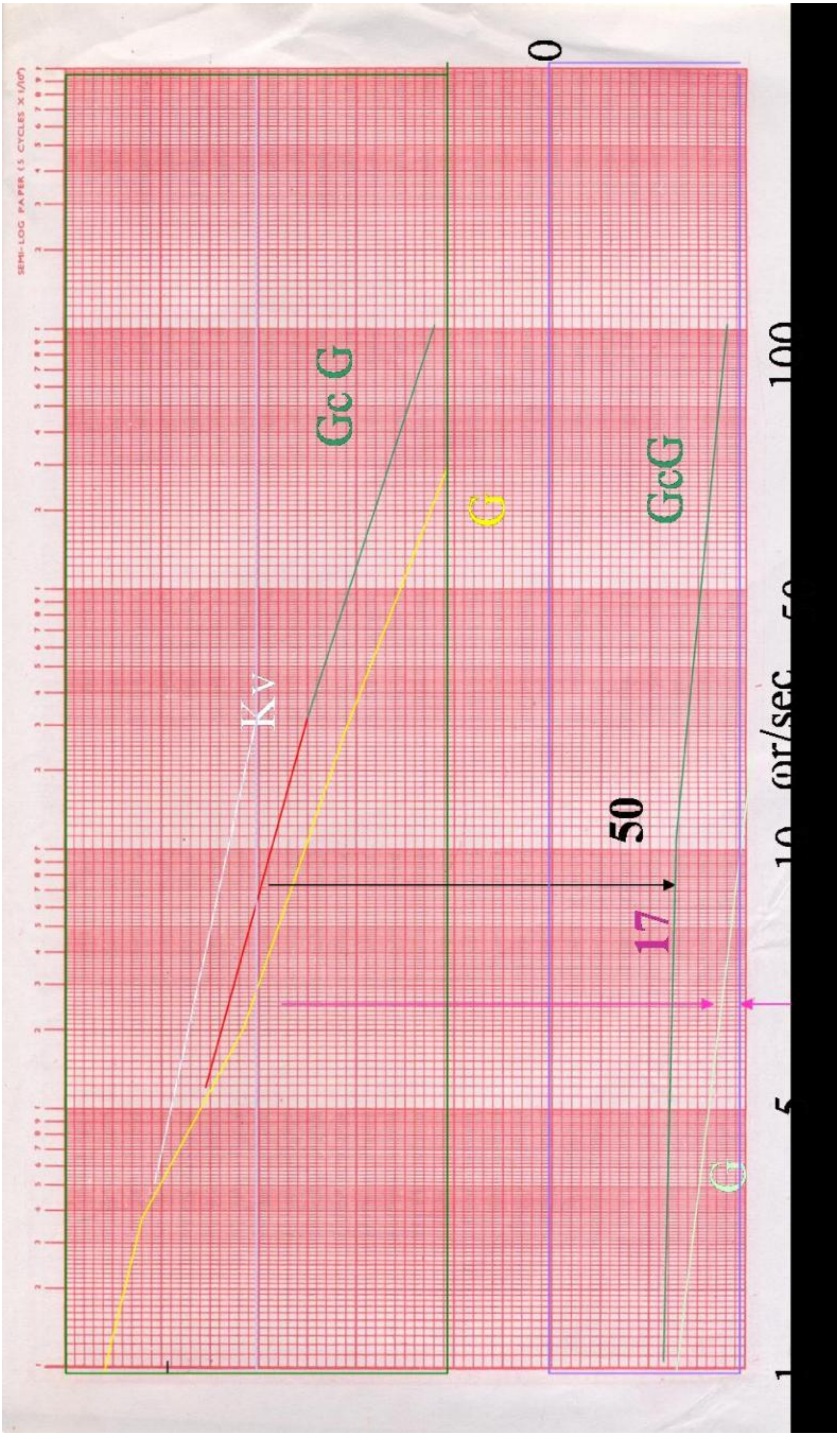
Amount of modification in the magnitude curve at ωm = [1/√ ά T] is

MOD [(1+j ωT)/(1+j ωά T)]

at ωm = 1/ √ά T

= { (1/√ ά)}= {(1/√0.24)}= 6.2 db

-6.2 corresponds to ω= 9 rad/sec which is new gain crossover frequency ωc


Gc G
G
Kv
50
17
GcG
G
0
100
ωr/sec
1

STEP 5

This frequency corresponds to

$\omega c = [1/\sqrt{\acute{\alpha}}\, T]$ is

(1/T) = $\sqrt{\acute{\alpha}}\omega$ = 4.41 and

(1/ $\acute{\alpha}$ T) = [$\omega c/\sqrt{\acute{\alpha}}$] = 18.4

Lead Network thus obtained is (S+4.41)/(S+18.4)

STEP 6

To compensate for the attenuation due to the lead network the amplifier GAIN is increased by a factor of [1/ $\acute{\alpha}$] = [1/ 0.24] = 4.17

Then TF of the compensator is

Gc(S) = (4.17) [(S+4.41)/(S+18.4)]

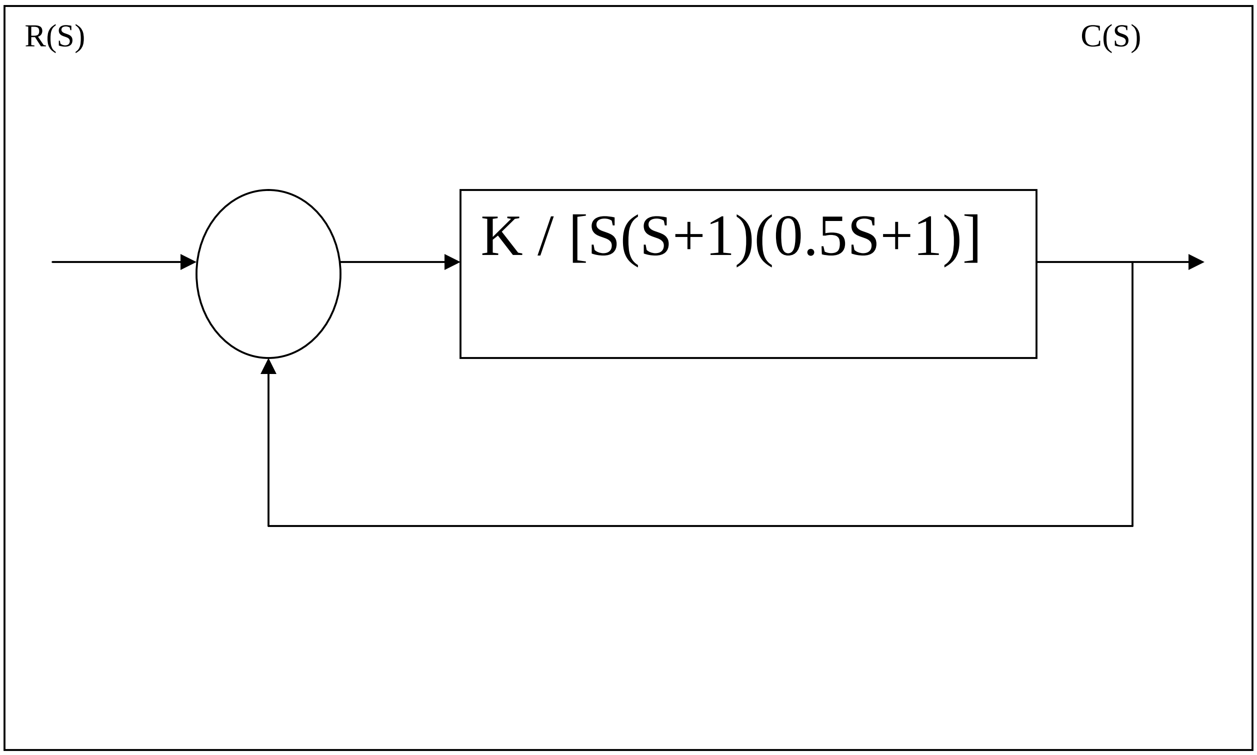
The compensated system has the OLTF as Gc(S) * G(S)

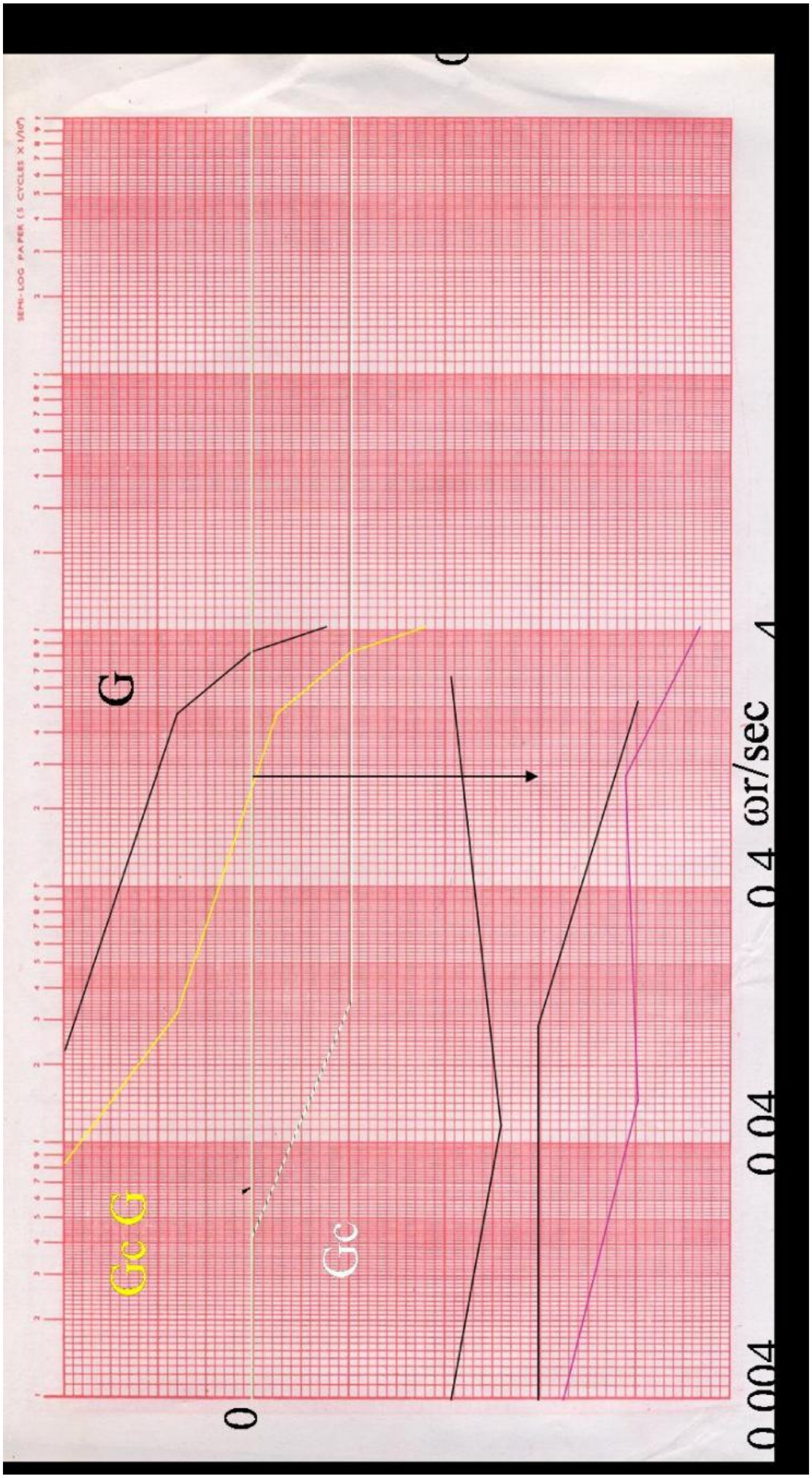
= (4.17) [(S+4.41)/(S+18.4)] *{40/S(S+2)}

ANALYSIS

- Lead Compensation causes the gain crossover frequency to increase from 6.3 to 9 rad/sec.
- Increase in frequency means increase in BANDWIDTH
- This increases the SPEED OF RESPONSE

PROBLEM - Design a suitable compensator for the system whose Open Loop Transfer Function is as shown in the block diagram. Static velocity error coefficient is 5 sec-1the phase margin is at least 40 and gain margin is at least 10 db.

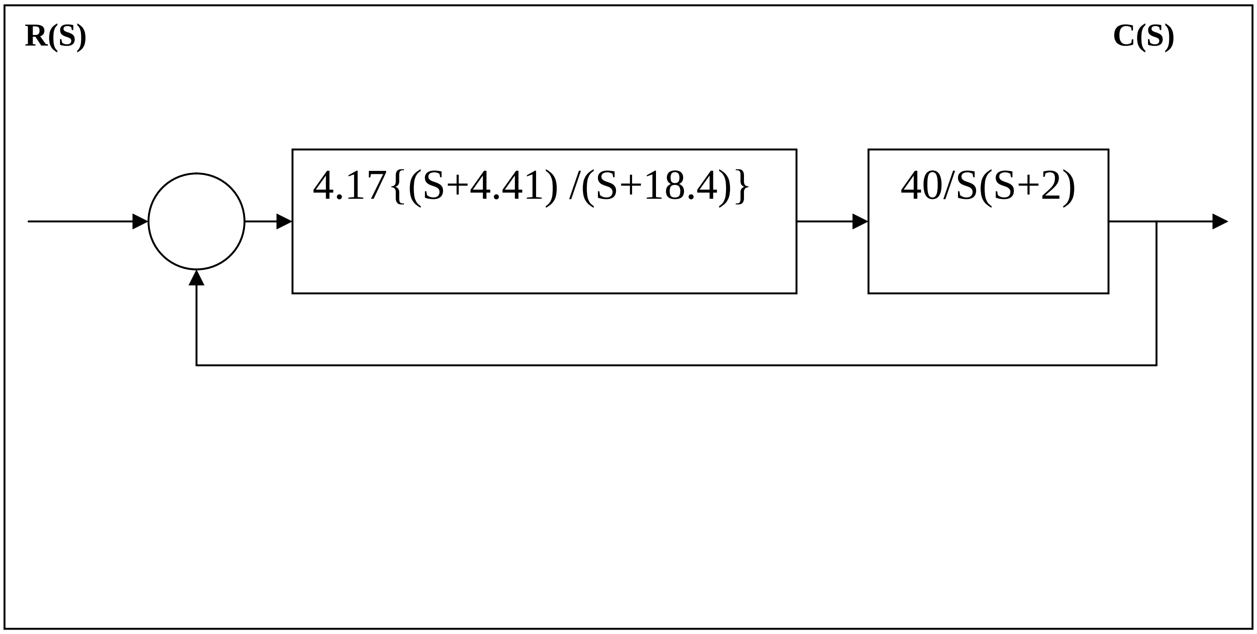


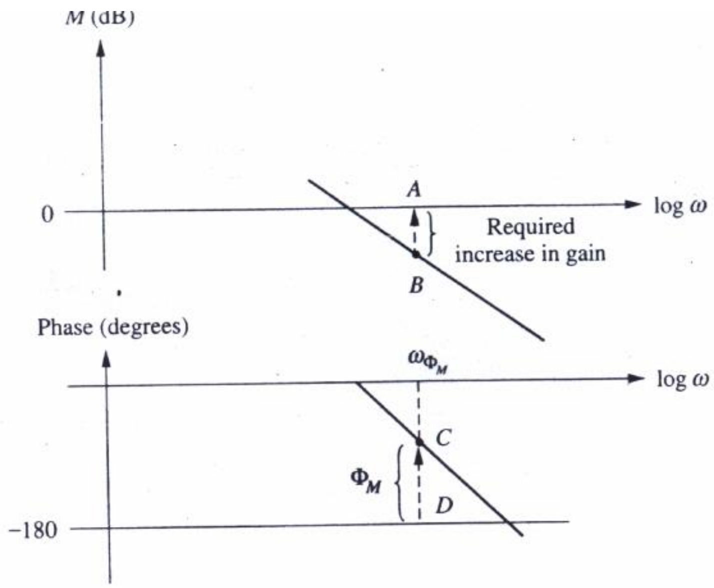

G
Gc G
Gc
0
0.004
0.04
0.4
4
ωr/sec

ANALYSIS

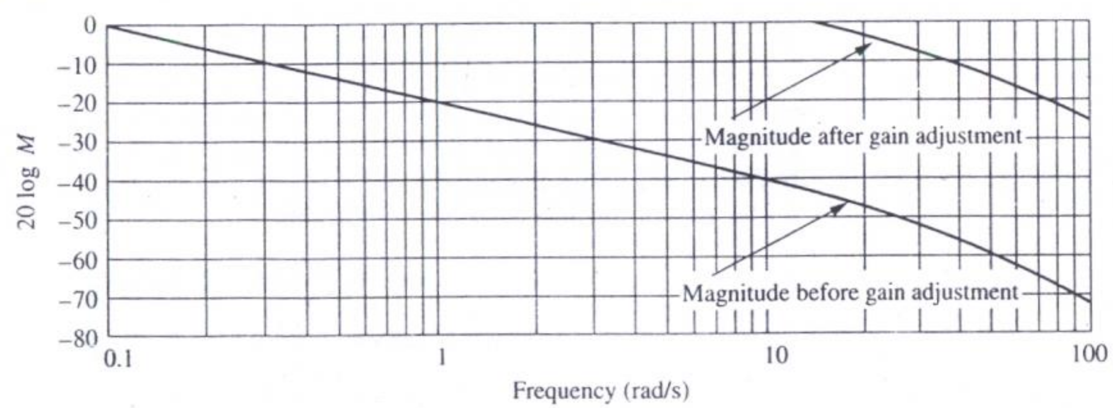
- Lag Compensation networks are low pass filters
- Permits high gain at low frequencies and reduces gain in the higher critical range
- Attenuation Characteristics of Lag network at high frequencies are utilised than phase lag characteristics
- Shift the gain cross over frequency to a lower frequency
- Decrease in frequency means decrease in BANDWIDTH
- This decreases the SPEED OF RESPONSE (transient)

COMPENSATED SYSTEM BLOCK DIAGRAM

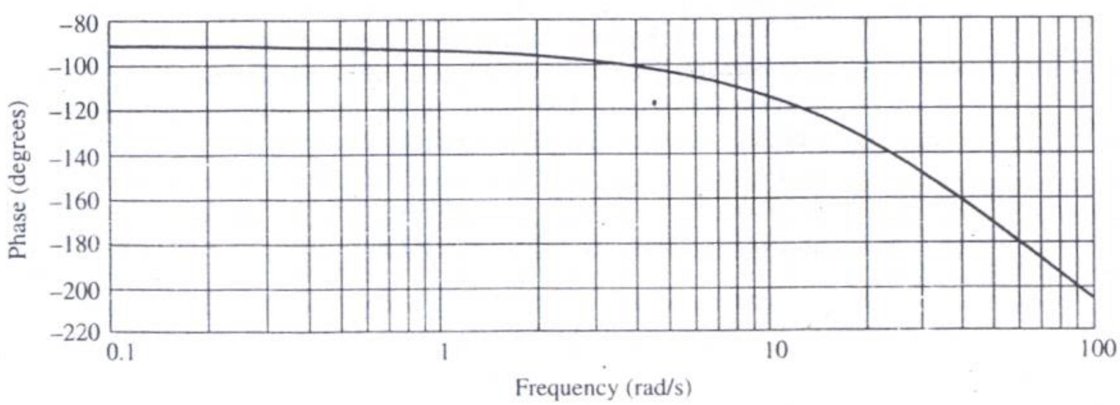




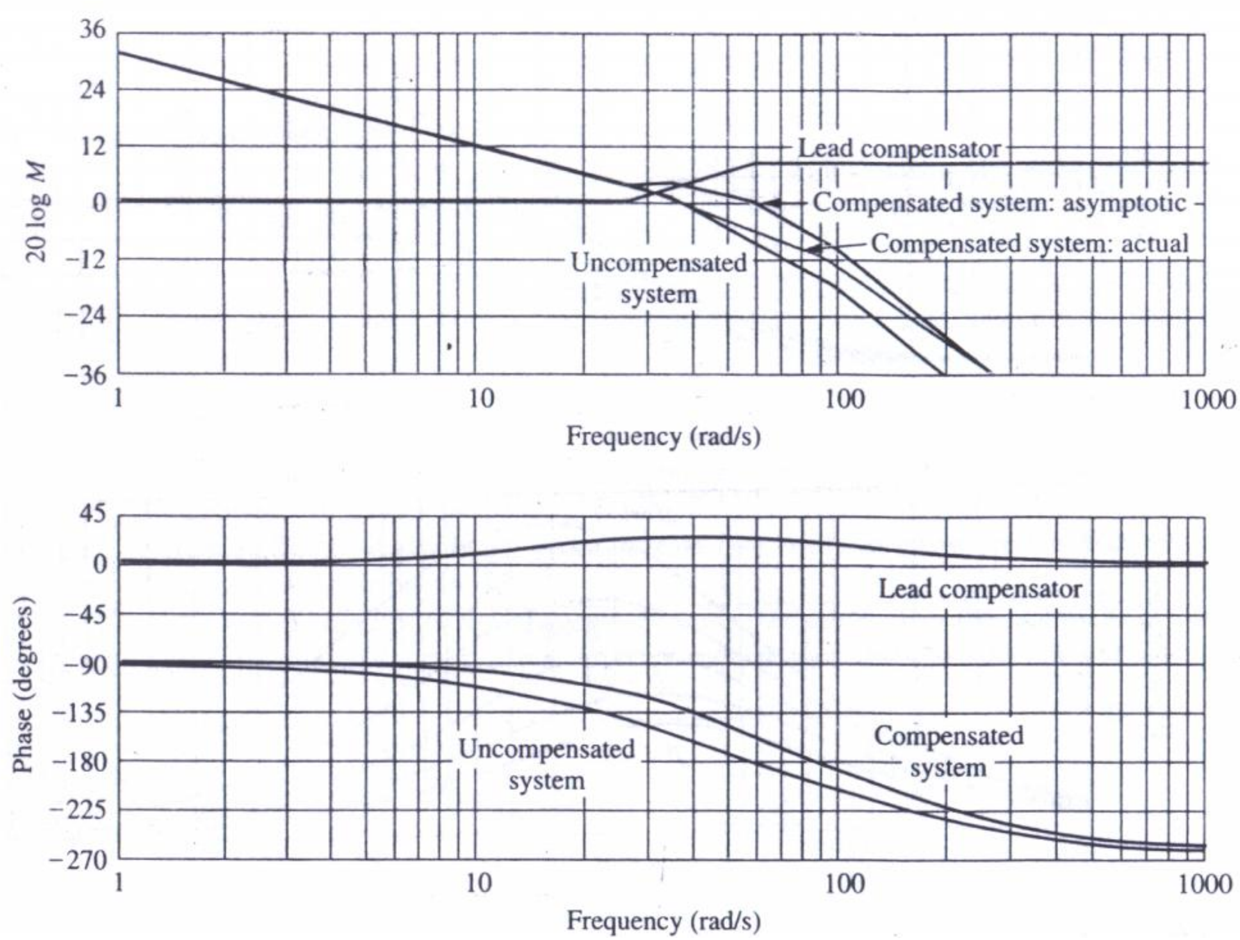
M (dB)
0
A
log ω
Required
increase in gain
B
Phase (degrees)
ω_{Φ_M}
log ω
C
Φ_M
D
−180

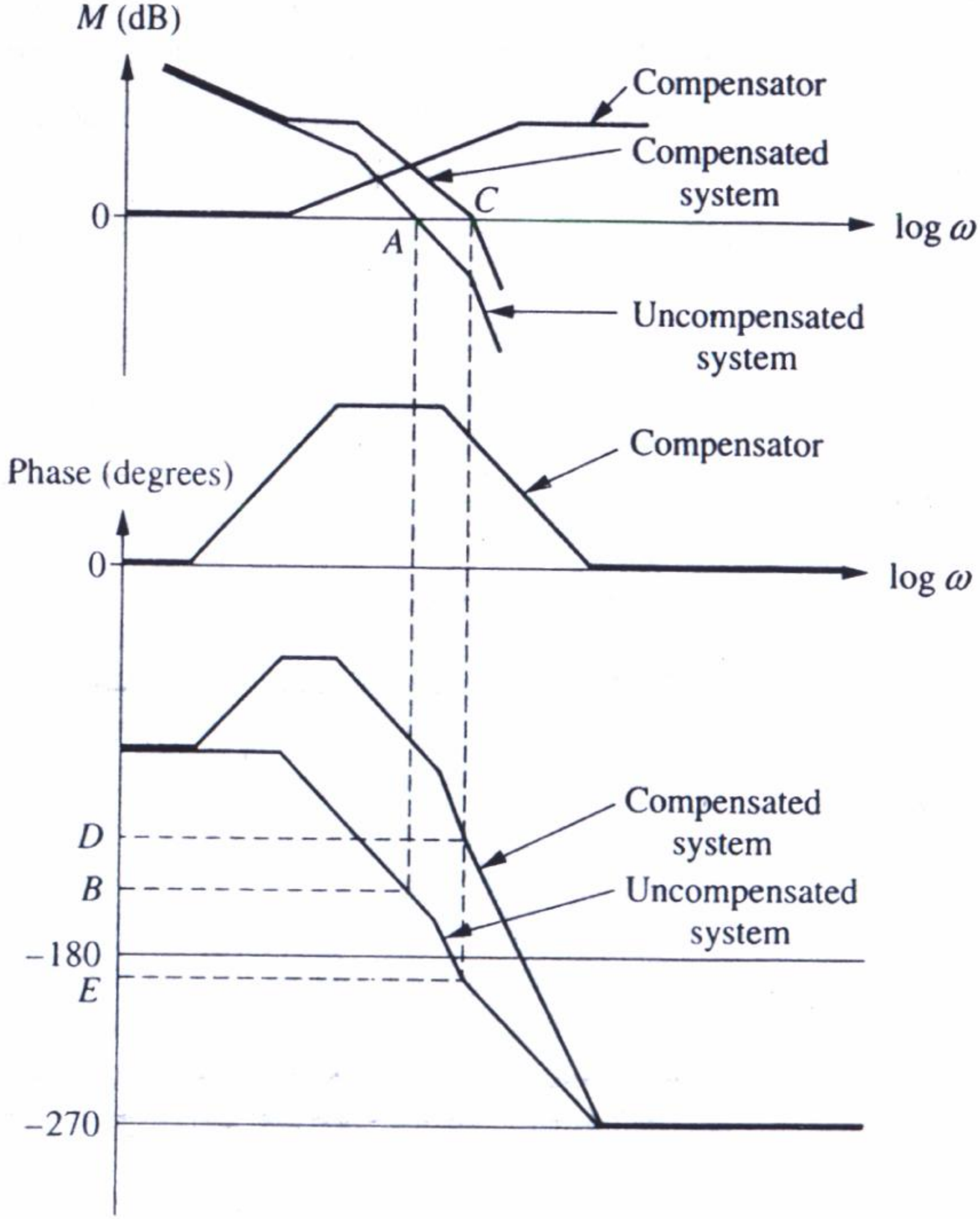


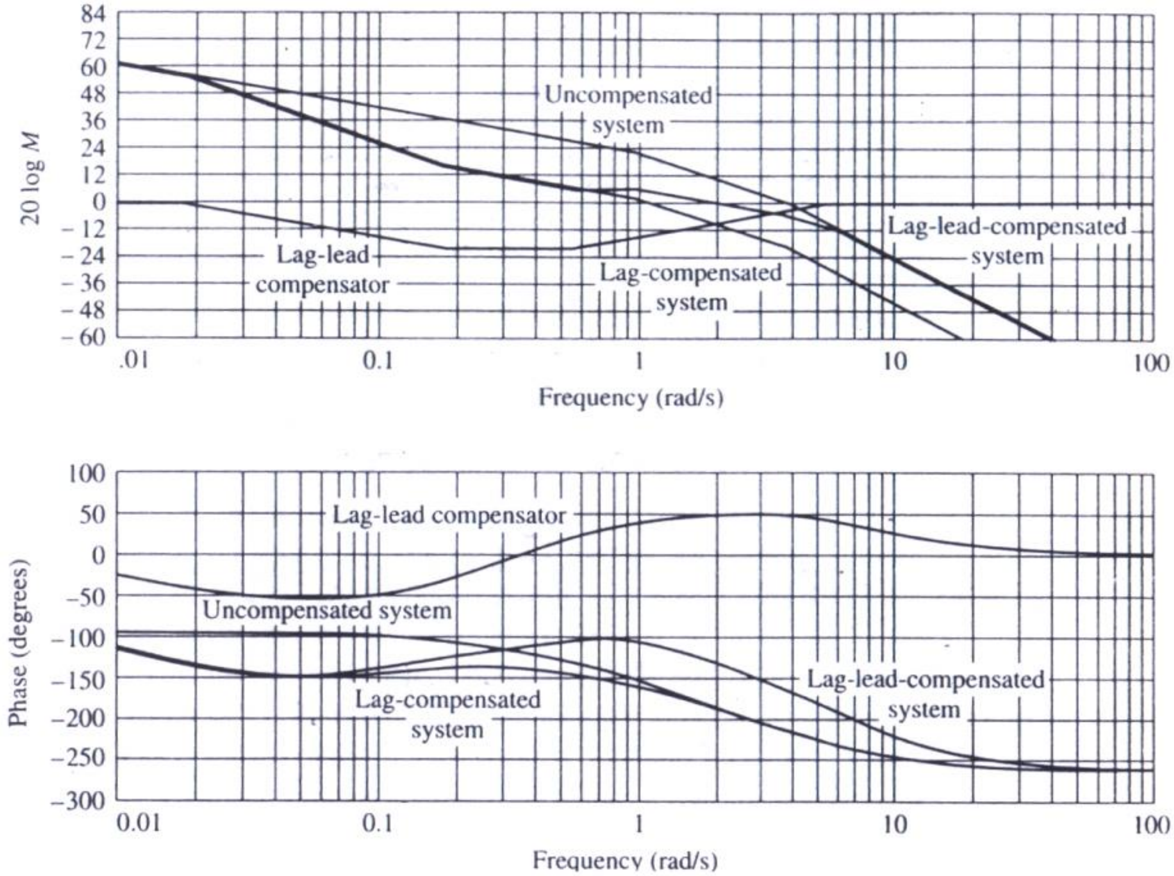
20 log M
0
−10
−20
−30
−40
−50
−60
−70
−80
Magnitude after gain adjustment
Magnitude before gain adjustment
0.1
1
10
100
Frequency (rad/s)



Phase (degrees)
−80
−100
−120
−140
−160
−180
−200
−220
0.1
1
10
100
Frequency (rad/s)


36
24
12
0
−12
−24
−36
20 log M
1
10
100
1000
Frequency (rad/s)
Lead compensator
Compensated system: asymptotic
Compensated system: actual
Uncompensated system
45
0
−45
−90
−135
−180
−225
−270
Phase (degrees)
1
10
100
1000
Frequency (rad/s)
Lead compensator
Compensated system
Uncompensated system


M (dB)
Compensator
Compensated
system
0
C
A
log ω
Uncompensated
system
Compensator
Phase (degrees)
0
log ω
Compensated
system
D
B
Uncompensated
system
−180
E
−270


84
72
60
48
36
24
12
0
−12
−24
−36
−48
−60
20 log M
Uncompensated system
Lag-lead-compensated system
Lag-lead compensator
Lag-compensated system
.01
0.1
1
10
100
Frequency (rad/s)
100
50
0
−50
−100
−150
−200
−250
−300
Phase (degrees)
Lag-lead compensator
Uncompensated system
Lag-compensated system
Lag-lead-compensated system
0.01
0.1
1
10
100
Frequency (rad/s)