UNIT- 2 & 3

SINGLE-PHASE TRANSFORMERS: Equivalent circuit, losses, efficiency, condition for maximum efficiency, all day efficiency. Open circuit and Short circuit tests, calculation of parameters of equivalent circuit. Regulation, predetermination of efficiency and regulation. Polarity test, Sumpner’s test. 6 Hours

**Losses in Transformer:**

Losses of transformer are divided mainly into two types:

1. Iron Loss
2. Copper Losses

**Iron Loss:**

This is the power loss that occurs in the iron part. This loss is due to the alternating frequency of the emf. Iron loss in further classified into two other losses.

a) Eddy current loss   b) Hysteresis loss

a) **EDDY CURRENT LOSS:** This power loss is due to the alternating flux linking the core, which will induced an emf in the core called the eddy emf, due to which a current called the eddy current is being circulated in the core. As there is some resistance in the core with this eddy current circulation converts into heat called the eddy current power loss. Eddy current loss is proportional to the square of the supply frequency.

b) **HYSTERESIS LOSS:** This is the loss in the iron core, due to the magnetic reversal of the flux in the core, which results in the form of heat in the core. This loss is directly proportional to the supply frequency.

Eddy current loss can be minimized by using the core made of thin sheets of silicon steel material, and each lamination is coated with varnish insulation to suppress the path of the eddy currents.
Hysteresis loss can be minimized by using the core material having high permeability.

**Copper Loss:**
This is the power loss that occurs in the primary and secondary coils when the transformer is on load. This power is wasted in the form of heat due to the resistance of the coils. This loss is proportional to the sequence of the load hence it is called the **Variable loss** where as the Iron loss is called as the **Constant loss** as the supply voltage and frequency are constants.

**Efficiency:**
It is the ratio of the output power to the input power of a transformer

\[
\text{Input} = \text{Output} + \text{Total losses} = \text{Output} + \text{Iron loss} + \text{Copper loss}
\]

Efficiency =

\[
\eta = \frac{\text{output power}}{\text{output power} + \text{Iron loss} + \text{Copper loss}} = \frac{V_2 I_2 \cos \phi}{V_2 I_2 \cos \phi + W_{iron} + W_{copper}}
\]

Where, \( V_2 \) is the secondary (output) voltage, \( I_2 \) is the secondary (output) current and \( \cos \Phi \) is the power factor of the load.

The transformers are normally specified with their ratings as KVA, Therefore,

\[
\text{Efficiency}; \eta = \frac{(\text{KVA}) \times 10^3 \times \cos \phi}{(\text{KVA}) \times 10^3 \times \cos \phi \times W_{iron} + W_{copper}}
\]

Since the copper loss varies as the square of the load the efficiency of the transformer at any desired load \( n \) is given by

\[
\text{Efficiency}; \eta = \frac{n \times (\text{KVA}) \times 10^3 \times \cos \phi}{n \times (\text{KVA}) \times 10^3 \times \cos \phi \times W_{iron} + n^2 \times W_{copper}}
\]
where $W_{\text{copper}}$ is the copper loss at full load

$$W_{\text{copper}} = I^2R \text{ watts}$$

**CONDITION FOR MAXIMUM EFFICIENCY:**

In general for the efficiency to be maximum for any device the losses must be minimum. Between the iron and copper losses the iron loss is the fixed loss and the copper loss is the variable loss. When these two losses are equal and also minimum the efficiency will be maximum.

Therefore the condition for maximum efficiency in a transformer is

$$\text{Copper loss} = \text{Iron loss}$$

(whichever is minimum)

**VOLTAGE REGULATION:**

The voltage regulation of a transformer is defined as the change in the secondary terminal voltage between no load and full load at a specified power factor expressed as a percentage of the full load terminal voltage.

$$\%\text{Voltage Regulation} = \frac{(\text{no load Sec. Voltage}) - (\text{full load Sec. Voltage})}{\text{full load Sec. Voltage}} \times 100$$

Voltage regulation is a measure of the change in the terminal voltage of a transformer between No load and Full load. A good transformer has least value of the regulation of the order of $\pm 5\%$

If a load is connected to the secondary, an electric current will flow in the secondary winding and electrical energy will be transferred from the primary circuit through the transformer to the load. In an ideal transformer, the induced voltage in the secondary winding ($V_s$) is in proportion to the primary voltage ($V_p$), and is given by the ratio of the number of turns in the secondary ($N_s$) to the number of turns in the primary ($N_p$) as follows:
Earlier it is seen that a voltage is induced in a coil when the flux linkage associated with the same changed. If one can generate a time varying magnetic field any coil placed in the field of influence linking the same experiences an induced emf. A time varying field can be created by passing an alternating current through an electric coil. This is called mutual induction. The medium can even be air. Such an arrangement is called air cored transformer.

Indeed such arrangements are used in very high frequency transformers. Even though the principle of transformer action is not changed, the medium has considerable influence on the working of such devices. These effects can be summarized as the followings.

1. The magnetizing current required to establish the field is very large, as the reluctance of the medium is very high.

2. There is linear relationship between the mmf created and the flux produced.

3. The medium is non-lossy and hence no power is wasted in the medium.

4. Substantial amount of leakage flux exists.

5. It is very hard to direct the flux lines as we desire, as the whole medium is homogeneous.

If the secondary is not loaded the energy stored in the magnetic field finds its way back to the source as the flux collapses. If the secondary winding is connected to a load then part of the power from the source is delivered to the load through the magnetic field as a link.

The medium does not absorb and lose any energy. Power is required to create the field and not to maintain the same. As the winding losses can be made very small by proper choice of material, the ideal efficiency of a transformer approaches 100%. The large magnetizing current requirement is a major deterrent.
1. Due to the large value for the permeance (μr of the order of 1000 as compared to air) the magnetizing current requirement decreases dramatically. This can also be visualized as a dramatic increase in the flux produced for a given value of magnetizing current.

2. The magnetic medium is linear for low values of induction and exhibits saturation type of non-linearity at higher flux densities.

3. The iron also has hysteresis type of non-linearity due to which certain amount of power is lost in the iron (in the form of hysteresis loss), as the B-H characteristic is traversed.

4. Most of the flux lines are confined to iron path and hence the mutual flux is increased very much and leakage flux is greatly reduced.

5. The flux can be easily ‘directed’ as it takes the path through steel which gives great freedom for the designer in physical arrangement of the excitation and output windings.
6. As the medium is made of a conducting material eddy currents are induced in the same and produce losses. These are called ‘eddy current losses’. To minimize the eddy current losses the steel core is required to be in the form of a stack of insulated laminations.

From the above it is seen that the introduction of magnetic core to carry the flux introduced two more losses. Fortunately the losses due to hysteresis and eddy current for the available grades of steel are very small at power frequencies. Also the copper losses in the winding due to magnetization current are reduced to an almost insignificant fraction of the full load losses. Hence steel core is used in power transformers.

In order to have better understanding of the behavior of the transformer, initially certain idealizations are made and the resulting ‘ideal’ transformer is studied. These idealizations are as follows:

1. Magnetic circuit is linear and has infinite permeability. The consequence is that a vanishingly small current is enough to establish the given flux. Hysteresis loss is negligible. As all the flux generated confines itself to the iron, there is no leakage flux.

2. Windings do not have resistance. This means that there are no copper losses, nor there is any ohmic drop in the electric circuit.

In fact the practical transformers are very close to this model and hence no major departure is made in making these assumptions. Fig 11 shows a two winding ideal transformer. The primary winding has $T_1$ turns and is connected to a voltage source of $V_1$ volts. The secondary has $T_2$ turns. Secondary can be connected to load impedance for loading the transformer. The primary and secondary are shown on the same limb and separately for clarity.

As a current $I_0$ amps is passed through the primary winding of $T_1$ turns it sets up an MMF of $I_0T_1$ ampere which is in turn sets up a flux through the core. Since the reluctance of the iron path given by $R = l/\mu A$ is zero as $\mu \rightarrow 1$, a vanishingly small value of current $I_0$ is enough to setup a flux which is finite. As $I_0$ establishes the field inside the transformer
it is called the magnetizing current of the transformer.

\[
\phi = \frac{mmf}{\text{Reluctance}} = \frac{I_0 T_1}{\frac{l}{\mu A}} = \frac{I_0 T_1 A \mu}{l}.
\]
This current is the result of a sinusoidal voltage $V$ applied to the primary. As the current through the loop is zero (or vanishingly small), at every instant of time, the sum of the voltages must be zero inside the same. Writing this in terms of instantaneous values we have, $v_1 - e_1 = 0$ where $v_1$ is the instantaneous value of the applied voltage and $e_1$ is the induced emf due to Faraday’s principle. The negative sign is due to the application of the Lenz’s law and shows that it is in the form of a voltage drop. Kirchoff’s law application to the loop will result in the same thing.

This equation results in $v_1 = e_1$ or the induced emf must be same in magnitude to the applied voltage at every instant of time. Let $v_1 = V_{1\text{peak}} \cos \omega t$ where $V_{1\text{peak}}$ is the peak value and $\omega = 2\pi f \ t$. $f$ is the frequency of the supply. As $v_1 = e_1$; $e_1 = v_1 \frac{d}{dt}$ but $e_1 = E_{1\text{peak}} \cos \omega t)$ $E_1 = V_1$. It can be easily seen that the variation of flux linkages can be obtained as $\psi_1 = \psi_{1\text{peak}} \sin \omega t$. Here $\psi_{1\text{peak}}$ is the peak value of the flux linkages of the primary.

Thus the RMS primary induced EMF is

$$e_1 = \frac{d\psi_1}{dt} = \frac{d(\psi_{1\text{peak}} \sin \omega t)}{dt} = \psi_{1\text{peak}} \omega \cos \omega t \quad \text{or the rms value}$$

$$E_1 = \frac{\psi_{1\text{peak}} \omega}{\sqrt{2}} \frac{2\pi f T_1 \phi_m}{\sqrt{2}} = 4.44 f \phi_m T_1 \quad \text{volts}$$

Here $\psi_{1\text{peak}}$ is the peak value of the flux linkages of the primary. The same mutual flux links the secondary winding. However the magnitude of the flux linkages will be $\psi_{2\text{peak}} = T_2 \phi_m$. The induced emf in the secondary can be similarly obtained as

$$e_2 = \frac{d\psi_2}{dt} = \frac{d(\psi_{2\text{peak}} \sin \omega t)}{dt} = \psi_{2\text{peak}} \omega \cos \omega t \quad \text{or the rms value}$$

$$E_2 = \frac{\psi_{2\text{peak}} \omega}{\sqrt{2}} \frac{2\pi f T_2 \phi_m}{\sqrt{2}} = 4.44 f \phi_m T_2 \quad \text{volts}$$
Transformer at loaded condition.

So far, an unloaded ideal transformer is considered. If now a load impedance $Z_L$ is connected across the terminals of the secondary winding a load current flows as marked in Fig. 11(c). This load current produces a demagnetizing mmf and the flux tends to collapse. However this is detected by the primary immediately as both $E_2$ and $E_1$ tend to collapse.

The current drawn from supply increases up to a point the flux in the core is restored back to its original value. The demagnetizing mmf produced by the secondary is neutralized by additional magnetizing mmf produces by the primary leaving the mmf and flux in the core as in the case of no-load. Thus the transformer operates under constant induced emf mode. Thus

$$i_1 T_1 - i_2 T_2 = i_0 T_1 \quad \text{but} \quad i_0 \to 0$$

$$i_2 T_2 = i_1 T_1 \quad \text{and the rms value} \quad I_2 T_2 = I_1 T_1.$$  

If the reference directions for the two currents are chosen as in the Fig. 12, then the above equation can be written in phasor form as,

$$\bar{I}_1 T_1 = \bar{I}_2 T_2 \quad \text{or} \quad \bar{I}_1 = \frac{T_2}{T_1} \bar{I}_2$$

Also

$$\frac{E_1}{E_2} = \frac{T_1}{T_2} = \frac{I_2}{I_1} \quad E_1 I_1 = E_2 I_2$$

Thus voltage and current transformation ratio are inverse of one another. If an impedance of $Z_L$ is connected across the secondary,

$$\bar{Z}_i = \frac{E_1}{I_1} = (\frac{T_1}{T_2})^2 \cdot \frac{E_2}{I_2} = (\frac{T_1}{T_2})^2 \cdot \bar{Z}_L$$
An impedance of $Z_L$ when viewed ‘through’ a transformer of turns ratio $(T_1/T_2)$ is seen as $(T_1/T_2)^2 \cdot Z_L$. Transformer thus acts as an impedance converter. The transformer can be interposed in between a source and a load to ‘match’ the impedance.

![Figure 13: Phasor diagram of Operation of an Ideal Transformer](image)

Finally, the phasor diagram for the operation of the ideal transformer is shown in Fig. 13 in which $\theta_1$ and $\theta_2$ are power factor angles on the primary and secondary sides. As the transformer itself does not absorb any active or reactive power it is easy to see that $\theta_1 = \theta_2$.

Thus, from the study of the ideal transformer it is seen that the transformer provides electrical isolation between two coupled electric circuits while maintaining power invariance at its two ends. However, grounding of loads and one terminal of the transformer on the secondary/primary side are followed with the provision of leakage current detection devices to safe guard the persons working with the devices. Even though the isolation aspect is a desirable one its utility cannot be over emphasized. It can be used to step up or step down the voltage/current at constant volt-ampere. Also, the transformer can be used for impedance matching. In the case of an ideal transformer the efficiency is 100% as there are no losses inside the device.
Practical Transformer

An ideal transformer is useful in understanding the working of a transformer. But it cannot be used for the computation of the performance of a practical transformer due to the non-ideal nature of the practical transformer. In a working transformer the performance aspects like magnetizing current, losses, voltage regulation, efficiency etc are important. Hence the effects of the non-idealization like finite permeability, saturation, hysteresis and winding resistances have to be added to an ideal transformer to make it a practical transformer.

Conversely, if these effects are removed from a working transformer what is left behind is an ideal transformer.

Finite permeability of the magnetic circuit necessitates a finite value of the current to be drawn from the mains to produce the mmf required to establish the necessary flux.

The current and mmf required is proportional to the flux density B that is required to be established in the core.

\[ B = \mu H; \quad B = \frac{\phi}{A} \]

where A is the area of cross section of the iron core m\(^2\). H is the magnetizing force which is given by,

\[ H = i \cdot \frac{T_1}{l} \]

where l is the length of the magnetic path, m. or

\[ \phi = B.A = \frac{A \mu(iT_1)}{l} = \text{permeance} \times \text{mmf (here that of primary)} \]

The magnetizing force and the current vary linearly with the applied voltage as long as the magnetic circuit is not saturated. Once saturation sets in, the current has to vary in a
nonlinear manner to establish the flux of sinusoidal shape. This non-linear current can be resolved into fundamental and harmonic currents. This is discussed to some extent under harmonics. At present the effect of this non-linear behavior is neglected as a secondary effect. Hence the current drawn from the mains is assumed to be purely sinusoidal and directly proportional to the flux density of operation. This current can be represented by a current drawn by an inductive reactance in the circuit as the net energy associated with the same over a cycle is zero. The energy absorbed when the current increases are returned to the electric circuit when the current collapses to zero. This current is called the magnetizing current of the transformer. The magnetizing current Im is given by \( Im = \frac{E1}{Xm} \) where Xm is called the magnetizing reactance. The magnetic circuit being lossy absorbs and dissipates the power depending upon the flux density of operation. These losses arise out of hysteresis, eddy current inside the magnetic core. These are given by the following expressions:

\[
P_h \propto B^{1.6} f
\]

\[
P_e \propto B^2 f^2 t^2
\]

- \( P_h \) - Hysteresis loss, Watts
- \( B \) - Flux density of operation Tesla.
- \( f \) - Frequency of operation, Hz
- \( t \) - Thickness of the laminations of the core, m.

For a constant voltage, constant frequency operation \( B \) is constant and so are these losses. An active power consumption by the no-load current can be represented in the input circuit as a resistance \( R_c \) connected in parallel to the magnetizing reactance \( X_m \). Thus the no-load current \( I_0 \) may be made up of \( I_c \) (loss component) and \( I_m \) (magnetizing component
as) $I_0 = I_c - j\mu I^2 cRc$ gives the total core losses (i.e. hysteresis + eddy current loss) $I^2mXm$ - Reactive volt amperes consumed for establishing the mutual flux.

Finite $\mu$ of the magnetic core makes a few lines of flux take to a path through the air. Thus these flux lines do not link the secondary winding. It is called as leakage flux. As the path of the leakage flux is mainly through the air the flux produced varies linearly with the primary current $I_1$. Even a large value of the current produces a small value of flux. This flux produces a voltage drop opposing its cause, which is the current $I_1$. Thus this effect of the finite permeability of the magnetic core can be represented as a series inductive element $jx_1$. This is termed as the reactance due to the primary leakage flux. As this leakage flux varies linearly with $I_1$, the flux linkages per ampere and the primary leakage inductance are constant (This is normally represented by $lI_1$ Henry). The primary leakage reactance therefore becomes $x_1 = 2\pi f lI_1$ ohm.

A similar effect takes place on the secondary side when the transformer is loaded. The secondary leakage reactance $jx_2$ arising out of the secondary leakage inductance $lI_2$ is given by $x_2 = 2\pi f lI_2$. Finally, the primary and secondary windings are wound with copper (sometimes aluminum in small transformers) conductors; thus the windings have a finite resistance (though small). This is represented as a series circuit element, as the power lost and the drop produced in the primary and secondary are proportional to the respective currents. These are represented by $r_1$ and $r_2$ respectively on primary and secondary side. A practical transformers ans these imperfections (taken out and represented explicitly in the electric circuits) is an ideal transformer of turns ratio $T_1 : T_2$ (voltage ratio $E_1 : E_2$). This is seen in Fig. 14. $I'_2$ in the circuit represents the primary current component that is required to flow from the mains in the primary $T_1$ turns to neutralize the demagnetizing secondary current $I_2$ due to the load in the secondary turns. The total primary current
(a) Physical arrangement
By solving this circuit for any load impedance $Z_L$ one can find out the performance of the loaded transformer.

The circuit shown in Fig. 14(b). However, it is not very convenient for use due to the presence of the ideal transformer of turns ratio $T_1 : T_2$. If the turns ratio could be made unity by some transformation the circuit becomes very simple to use. This is done here by replacing the secondary by a ‘hypothetical’ secondary having $T_1$ turns which is ‘equivalent’ to the physical secondary. The equivalence implies that the ampere turns, active and reactive power associated with both the circuits must be the same. Then there is no change as far as their effect on the primary is considered.

vectorially is $\bar{I}_1 = \bar{I}_2 + \bar{I}_0$

Here $I_2' T_1 = I_2 T_2$ or $I_2' = I_2 \frac{T_2}{T_1}$

Thus $\bar{I}_1 = \bar{I}_2 \frac{T_2}{T_1} + \bar{I}_0$
Thus

\[ V'_2 = aV_2, \quad I'_2 = \frac{I_2}{a}, \quad r'_2 = a^2 r_2, \quad x'_l = a^2 x_l, \quad Z'_L = a^2 Z_L. \]

where a -turns ratio \( T_1/T_2 \)

As the ideal transformer in this case has a turns ratio of unity the potentials on either side are the same and hence they may be conductively connected dispensing away with the ideal transformer. This particular equivalent circuit is as seen from the primary side. It is also possible to refer all the primary parameters to secondary by making the hypothetical equivalent primary winding on the input side having the number of turns to be \( T_2 \). Such an equivalent circuit having all the parameters referred to the secondary side is shown in fig.

The equivalent circuit can be derived, with equal ease, analytically using the Kirchoff’s equations applied to the primary and secondary. Referring to fig. 14(a), we have (by neglecting the shunt branch)

\[
\begin{align*}
V_1 &= E_1 + I_1(r_1 + jx_1) \\
E_2 &= V_2 + I_2(r_2 + jx_2) \\
T_1 I_0 &= T_1 I_1 + T_2 I_2 \quad \text{or} \quad I_1 = -\frac{I_2}{a} + I_0 \\
I_0 &= -\frac{I_2}{a} + I_c + I_m \\
\alpha &= \frac{T_1}{T_2}.
\end{align*}
\]

Multiply both sides of Eqn.34 by ‘a’ [This makes the turns ratio unity and retains the power invariance].

\[ aE_2 = aV_2 + aI_2(r_2 + jx_2) \quad \text{but} \quad aE_2 = E_1 \]
Substituting in Eqn we have

\[
V_1 = aV_2 + aI_2(r_2 + jx_{l2}) + I_1(r_1 + jx_{l1}) \\
= V'_2 + I_1(a^2r_2 + ja^2x_{l2}) + I_1(r_1 + jx_{l1}) \\
= V'_2 + I_1(r_1 + r'_2 + jx_{l1} + x'_{l2})
\]

A similar procedure can be used to refer all parameters to secondary side. (Shown in fig)