06IP/ IM74 OPERATIONS RESEARCH

In order to develop a general solutions for LPP having more than 2 variables the LPP must be put in the standard form.

The ideas conveyed by the graphical LP solution lay the foundation for the development of the algebraic simplex method.

Graphical Method	Algebraic simplex method
Solution space consists of infinity of feasible points	The system has infinity of feasible solution
Candidate for the optimum solution are given by a finite number of corner points	Candidate for the optimum solution are given by a finite number of basic feasible solution
Use the objective function to determine the optimum corner point	Use the objective function to determine the optimum basic feasible solution

Principles of the simplex method.

The simplex method is developed by G.B Dantzig is an iterative procedure for solving linear programming problem expressed in standard form. In addition to this simplex method requires constraint equations to be expressed as a canonical system form which a basic feasible solution can be readily obtained

The solutions of any LPP by simplex algorithm the existence of an IBFS is always assumed, the following steps help to reach an optimum solution of an LPP.

Procedure for Maximization

Step 1 write the given LPP in standard form

Step 2 check whether the objective function of the given LPP is to be Maximized or minimized if its to be minimized then we convert it into a problem of maximizing using the result.

Min z = - Max Z or (-Z)

Step 3 check whether all b_i (i=123…..m0 are non-negative. if anyone b_i is –VE, then multiply the corresponding in equation of the constraint by -1.

Step 4 Convert all the in equations of the constraint into equations by introducing slack or surplus variable in the constraints. Put these costs equal to zero in objective cost.

Step 5 Obtain an IBFS to problem in the form identity matrix form in canonical form
$$\begin{bmatrix} 1\ 0\ 0 \\ 0\ 1\ 0 \\ 0\ 0\ 1 \end{bmatrix}$$ and put it in the 1st column of simples table.

Step 6 compute the net evaluation row $(Z_j – C_j)$ (j=1,2,3….n)
$Z_j – C_j$ = P/U (Profit / unit) x X_j -C_j (j=1,2,3…n)
examine the sign of $Z_j – C_j$
i)if all $(Z_j – C_j)$ >= 0 then the IBFS solutions column is an optimum basic feasible solutions.
ii) if at least one $(Z_j – C_j)$ <=0 proceed to the next step

Step 7 if there are more than one –ve $(Z_j – C_j)$ then choose the most –ve of them then it will become key column
i)if all the no's in the key column is –ve then there is an unbounded solutions to the given problem
ii)if at least one X_m >0 (m =123…n) then the corresponding vector X_m >=0 (m=1,2,3…n) then the corresponding vector X_m entry the basis of solution column

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Step 8 compute the ratios = solutions column no / key column no.

And choose the minimum of them. Let the minimum of these ratios be the key row these variable in the basic variable column of the key row will become the leaving element or variable.

Step 9 using the below relation to find new no of other than key row and new no for key row also

New no for pivot row = current pivot row / pivot element

Other than key row

New element = old element – (PCE * NPRE)

PCE= Pivot column element, NPRE=new pivot row element

New no= old no – (corresponding Key column / Key element)

x (corresponding key row)

Step 10 go to step5 and repeat the computational procedure until either an optimum solutions is obtained or there is an indication of an unbounded solution.

Note : case 1 in case of a tie for entering basis vector. i.e., there are 2 or more $Z_j – C_j$ which are equal and at the same time the highest –ve values then arbitrary selection of any one of them will not alter optimality.

Case 2 in case of a tie for the leaving variable i.e., there are 2 or more min ratio column i.e., (solution no / key column no) which are equal and greater than zero then arbitrary select any one of them will not alter optimality. But, if the tied ratios are zeros then charnes method of penalty should be followed.

Problem 1:

Use simplex method to solve the given LPP

Max Z $= 5x_1 + 3x_2$

STC

$x_1 + x_2 <= 2$

$5x_1 + 2x_2 <= 10$

$3x_1 + 8x_2 <= 12$

$x_1, x_2 >= 0$

Solution:

Step 1: Since the problem is maximization problem all the constraint are $<=$ type and the requirements are +ve. This satisfies the simplex method procedure.

Step 2: since all the constraints are $<=$ type we introduce the slack variables for all the constraints as $x_3 >= 0$, $x_4 >= 0$, $x_5 >= 0$ for the I II and III constraint

Step 3: the given LPP can be put in standard form

Max Z $= 5x_1 + 3x_2 + (0) x_3 + (0) x_4 + (0)x_5$

STC

$x_1 + x_2 + x_3 <= 2$

$5x_1 + 2x_2 + x_4 <= 10$

$3x_1 + 8x_2 + x_5 <= 12$

$x_1, x_2, x_3, x_4, x_5 >= 0$

Step 4: matrix form

Max Z $= (5,3,0,0,0)$ $(x_1, x_2, x_3, x_4, x_5)$

STC

$$\begin{Bmatrix} 1 & 1 & 1 & 0 & 0 \\ 5 & 2 & 0 & 1 & 0 \\ 3 & 8 & 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 10 \\ 12 \end{Bmatrix}$$

$x_1, x_2, x_3, x_4, x_5 >= 0$

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Step 5 : since, considering sub-matrix from the matrix are which form basic variables for the starting table of simplex

(1 0 0) (0 1 0) (0 0 1) are linearly independent column vectors of A.

Therefore, the sub Matrix is

$$B= \begin{Bmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{Bmatrix}$$

The corresponding variables of the sub matrix is (x_3, x_4, x_5) and these variables are the basic variables for the starting iteration of the simplex problem and there obvious initial basic feasible solutions are $(x_3, x_4, x_5) = (2, 10, 12)$

			5	3	0	0	0	
Basic Variable (V)	Profit / Unit (P/U)	solution	x_1	x_2	x_3	x_4	x_5	Min Ratio = soln. no /key column No.
	0	2	1	1	1	0	0	2/1 = 2
x_3	0	10	5	2	0	1	0	10/5 = 2
x_5	0	12	3	8	0	0	1	12/3 = 4
	Max Z= 0x2+0x10 +0x12		=0x1 +0x5 +0x3 -5	=0x1 +0x2 +0x8 -3	=0x1 +0x0 +0x0 -0	=0x0 +0x1 +0x0 -0	=0x0 +0x0 +0x1 -0	
			-5	-3	0	0	0	

Leaving variable

Key Row

Key Column

Entering Variable

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| | | | 5 | 3 | 0 | 0 | 0 | |
Basic Variable (B V)	Profit / Unit (P/U)	solution	x_1	x_2	x_3	x_4	x_5	Min Ratio = soln. no /key column No.
x_1	5	2	1	1	1	0	0	
x_4	0	0	0	-3	-5	1	0	
x_5	0	6	0	5	-3	0	1	
	Max Z= 5x2+0x0+0x6 =10		=5x1+ 0x0 +0x0- 5	=5x1+0 x-3 +0x5 -3	=5x1 +0x-5 + 0x-3 - 0	=5x0+ 0x1+0 x0-0	=5x0+ 0x0+0 x1-0	
			0	2	5	0	0	

New Numbers for Key row
Soln.= old no / Key element
$\quad$ = 2/ =2
x_1 $\quad$ = 1/1 = 1
x_2 $\quad$ = 1/1 = 1
x_3 $\quad$ = 1/1 = 1
x_4 $\quad$ = 0/1 = 0
x_5 $\quad$ = 0/1 = 0

other than key rows new no is
found by using the following
formulae
New No=old element – PCE*NPRE
for x_4 row new no are
Soln. = 10-5*2= 0
x_1 = 5-5*1 = 0
x_2 = 2-5*1 = -3
x_3 = 0-5*1 = -5
x_4 = 1-5*0 =1
x_5 = 0 -5*0 =0
for x_5 row the new no are
Soln. = 12-3*2 =6
x_1 =3-3*1 = 0
x_2 = 8-3*1 = 5
x_3 = 0-3*1 = -3
x_4 = 0-3*0 = 0
x_5 = 1 -3*0 = 1

Since, the given problems net evaluation row is +ve, then given problem as attained the optimum

Therefore, $x_1 = 2$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$, $x_5 = 6$,
Substitute in the objective function

Max $Z = 5x_1 + 3x_2 + (0) x_3 + (0) x_4 + (0)x_5$
Max $Z = 5 \times 2 + 3 \times 0 + 0 \times 0 + 0 \times 0 + 0 \times 6$
Max $Z = 10$

Problem no 2

Solve the given problem by simplex method
Max $Z = 107x_1 + x_2 + 2 x_3$
STC
$14x_1 + x_2 - 6x_3 + 3x_4 = 7$
$16x_1 + 1/2x_2 - 6x_3 \leq 5$
$16x_1 - 8x_2 - x_3 \leq 0$
$x_1, x_2, x_3, x_4 \geq 0$

Solutions:

In the given problem the objective function is MaxZ and it has only three variables.
The I constraint is of standard form already slack variable is introduced as $x_4 \geq 0$ and the value should be one but, it is having 3 due this it should be divided by three for enter equation on both sides and II & III constraint are of $\leq$ type so we introduce $x_5 \geq 0$ $x_6 \geq 0$ as slack variable.
Then the given problem can be rewritten as
Max $Z = 107x_1 + x_2 + 2 x_3 + 0x_4 + 0 x_5 + 0 x_6$
STC
$14/3x_1 + 1/3 x_2 - 6/3x_3 + 3/3x_4 = 7/3$
$16x_1 + 1/2x_2 - 6x_3 + x_5 = 5$
$16x_1 - 8x_2 - x_3 + x_6 = 0$
$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

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The matrix form

Max Z= (107, 1, 2, 0, 0, 0) (x_1, x_2, x_3, x_4, x_5, x_6)

STC

$$
\begin{Bmatrix}
14/3 & 1/3 & -2 & 1 & 0 & 0 \\
16 & 1/2 & -6 & 0 & 1 & 0 \\
16 & -1 & -1 & 0 & 0 & 1
\end{Bmatrix}
\begin{Bmatrix}
x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6
\end{Bmatrix}
=
\begin{Bmatrix}
7/3 \\ 5 \\ 0
\end{Bmatrix}
$$

$x_1, x_2, x_3, x_4 \ x_5 \ x_6 \ >= 0$

$$
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

Now, the canonical form from the above matrix is
the corresponding variables are ($x_4 \ x_5 \ x_6$) and their
obvious solution is (7/3 5 0)

Starting Table:

			107	1	2	0	0	0	
Basic variable (B.V)	Profit /unit P/U	Solution	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio =Soln. no/ Pivot column no
x_4	0	7/3	14/3	1	-6	1	0	0	=7/3 / 14/3=0.5
x_5	0	5	16	½	-6	0	1	0	=16/5= 3.2
x_6	0	0	16	-1	-1	0	0	1	=16/0=∞
	Max Z = 0 x 7/3 + 0 x 5 + 0 x 0 = 0		=(0x14/3 +0x16+0x 16) -107	=(0x1+0 x1/2+0x -1) -1	= (0 x-6 +0x- 6+0x-1) -2	=(0x1 +0x0 +0x0) - 0	=(0x0+ 0x1+0x 0)- 0	=(0x0+0 x0+0x1) - 0	
			-107	-1	-2	0	0	0	

First Table 1:

| | | | 107 | 1 | 2 | 0 | 0 | 0 | |
Basic variable (B.V)	Profit /unit P/U	Solution	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio =Soln. no/ Pivot column no
x_1	107	0.5	1	3/14	1	3/14	0	0	
x_5	0	-3	0	-41/14	-22	-48/14	1	0	
x_6	0	-8	0	-62/14	- 17	-48/14	0	1	
	Max Z = 107 x 0.5 + 0 x -3 +0 x -8 = 0		=(1x107 +0x0 +0x0) -107	=(107x3 /14 + 0 x -41/14 + 0 x -62/14) -1	= (107 x 1 + 0 x -22 + 0 x-17) -2	=(107x3 /14 + 0 x -48/14 + 0 x -48/14) - 0	=(107 x 0 +0 x 1+ 0 x 0) - 0	=(107x0 + 0x0 +0x1) - 0	
	Max Z=53.5	0	+ve	+ve	+ve	+ve	0		

New Numbers for Key row

Soln.= old no / Key element

$= 7/3/ 14/3 =0.5$

$x_1 = 14/3 / 14/3 = 1$

$x_2 = 1/14/3 = 3/14$

$x_3 = -6 / 14/3 = 1$

$x_4 = 1/14/3 = 3/14$

$x_5 = 0/14/3 = 0$

$x_6 = 0/14/3 = 0$

for x_6 row the new no are

Soln. $= 0-16*0.5 = -8$

$x_1 =16-16*1 = 0$

$x_2 = -1-16*3/14 = -62/14$

$x_3 = -1-16*1 = -17$

$x_4 = 0-16*3/14 = -48/14$

$x_5 = 0 -16*0 = 0$

$x_6 = 1 -16*0 = 1$

other than key rows new no is

found by using the following

formulae

New No=old element – PCE*NPRE

for x_5 row new no are

Soln. $= 5-16*0.5= -3$

$x_1 = 16-16*1 = 0$

$x_2 = 1/2-16*3/14 = -41/14$

$x_3 = -6-16*1 = -22$

$x_4 = 0-16*3/14= - 48/14$

$x_5 = 1 -16*0 =1$

$x_6 = 0 -16*0 =0$

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Since, all the NER is positive then given problem is optimal

Therefore, $x_1 = 0.5$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$, $x_5 = -3$, $x_6 = -8$

Max Z = 107*0.5 + 1*0 + 2*0 + 0*0 + 0*-3 + 0* -8

       = 53.5

Problem 3:

Solve the given LPP by simplex method

Max $Z = 4x_1 + 5x_2 + 9x_3 + 11x_4$

STC

$x_1 + x_2 - 6x_3 + 3x_4$ ≤ 7

$7x_1 + 5x_2 + 3x_3 + 2x_4$ ≤ 120

$3x_1 + 5x_2 + 10x_3 + 15x_4$ ≤ 100

$x_1, x_2, x_3, x_4 \geq 0$

Solution:

The given problem is maximization problem and all the constraints are <= type, so, we introduce the slack variable as $x_5 \geq 0$,

$x_6 \geq 0$, $x_7 \geq 0$ for I , II and III constraints respectively.

The given LPP can be written as

Max $Z = 4x_1 + 5x_2 + 9x_3 + 11x_4$

STC

$x_1 + x_2 + x_3 + x_4 + x_5$ $= 7$

$7x_1 + 5x_2 + 3x_3 + 2x_4 + x_6$ $= 120$

$3x_1 + 5x_2 + 10x_3 + 15x_4 + x_7$ $= 100$

$x_1, x_2, x_3, x_4 \geq 0$

Matrix form

Max Z = (4, 5, 9, 11, 0, 0, 0) (x_1, x_2, x_3, x_4 x_5, x_6, x_7)

STC

$$\begin{Bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 7 & 5 & 3 & 2 & 0 & 1 & 0 \\ 3 & 5 & 10 & 15 & 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 120 \\ 100 \end{Bmatrix}$$

x_1, x_2, x_3, x_4 x_5, x_6, x_7 >= 0

Starting table

Basic variable	Profit / unit	solution	x_1	x_2	x_3,	x_4	x_5	x_6	x_7	Min ratio
			4	5	9	11	0	0	0	
x_5	0	7	1	1	1	1	1	0	0	7/1
x_6	0	120	7	5	3	2	0	1	0	120/2
x_7	0	100	3	5	10	15	0	0	1	100/15
	MaxZ = 0x7+0x120 +0x100		-4	-5	-9	-11	0	0	0	

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BIG M Method or Methods of Penalties

Whenever the objective function is MinZ and when all or some of the constraints are of >= type or = type. We introduce surplus variable and a artificial variable to LHS of the constraint when it is necessary to complete the identity matrix I.

The general practice is to assign the letter M as the cost in a minimization problem, and – M as the profit in the maximization problem with assumption that M is a very large positive number to the artificial variables in the objective function.

The method of solving a LPP in which a high penalty cost has been assigned to the artificial variables is known as the method of penalties or BIG m Method.

Procedures

Step1: At any iteration of the usual simplex method can arise any one of the following three cases:

Case a) if there is no vector corresponding to some artificial variable in the solution column in such case, we proceed to step 2.

Case b) if at least one vector corresponding to some artificial variable, in the basis is basic variable column at the zero level i.e., corresponding entry in solution column is zero and the co-efficient of m in each net evaluation Z_j - C_j is non negative.

In such case, the current basic feasible solution is a degenerate one.
If this is a case when an optimum solution. The given LPP includes an artificial basic variable and an optimum basic feasible solution does not exist.

Case c) if at least one artificial vector is in the basis Y_b but, not at zero level i.e., the corresponding entry in X_b is non zero. Also co-efficient of M in each net evaluation Z_j - C_j is non negative.

In the case, the given LPP does not possess an optimum basic feasible solution. Since, M is involved in the objective function. In such case, the given problem has a pseudo optimum basic feasible solution.

Step 2: application of simplex method is continued until either an optimum basic feasible solution is obtained or there is an indication of the existence of an unbounded solution to the given LPP.

Note: while applying simplex method, whenever a vector corresponding to some artificial variable happens to leave the basis, we drop that vector and omit all the entries corresponding to that vector from the simplex table.

Problem 1:

Solve the give LPP by BIG M Method

Max $Z = 3x_1 - x_2$

STC

$2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$8x_2 \leq 4$

$x_1, x_2 \geq 0$

Since, the given problem is max z and the I constraint is $\geq$ type we introduce surplus variable as $x_3 \geq 0$, and a artificial variable as $x_4 \geq 0$, the II & III constraint are of $\leq$ type, so we introduce $x_5 \geq 0$, $x_6 \geq 0$.

The standard of the given LPP as follow as.

Max $Z = 3x_1 - x_2 + 0x_3 - Mx_4 + 0x_5 + 0x_6$

STC

$2x_1 + x_2 - x_3 + x_4 \qquad = 2$

$x_1 + 3x_2 + x_5 \qquad = 3$

$\qquad x_2 + x_6 \qquad = 4$

$x_1, x_2, x_3, x_4, x_5, x_6 >= 0$

The Matrix form

Max $Z = (3, -1, 0, -M, 0, 0) (x_1, x_2, x_3, x_4, x_5, x_6)$

STC

$$\begin{Bmatrix} 2 & 1 & -1 & 1 & 0 & 0 \\ 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 3 \\ 4 \end{Bmatrix}$$

$x_1, x_2, x_3, x_4, x_5, x_6 >= 0$

$$\begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$ (x_4, x_5, x_6) canonical system.

The basic variable and their obvious solution

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Starting table:

			3	-1	0	-M	0	0	
Basic Variable	Profit / unit	solution	x₁	x₂	x₃	x₄	x₅	x₆	Min ratios
x_4	-M	2	2	1	-1	1	0	0	2/2=1
x_5	0	3	1	3	0	0	1	0	3/1=3
x_6	0	4	0	8	0	0	0	1	4/0=∞
	MaxZ= -mx2+0x3 +0x4=-2m		=-mx2+0x1+0x4 -3= -2m-3	=-mx1+0x3+0x0-(-1)= -m+1	=-mx-1+0x0 +0x0-0= m	=-mx1+0x0+0x0-)-m)=0	=-mx0+0x1+0x0-0=0	=-mx0+0x0+0x1-0=0	

			3	-1	0	-M	0	0	
Basic Variable	Profit / unit	solution	x₁	x₂	x₃	x₄	x₅	x₆	Min ratios
x_1	3	1	1	½	-1/2	1/2	0	0	1/-1/2=-2
x_5	0	2	0	5/2	1/2	-1/2	1	0	2/1/2=4
x_6	0	4	0	1	0	0	0	1	4/0=∞
	MaxZ=3x1+0x2+0x4=3		=3x1+0x0+0x0 – 3	3x1/2+0x5/2+0x1-(-1)	=3x-1/2+0x1/2+0x0-0	=3x1/2+ox-1/2+0x0 – (-M)	=3x0+0x1+0x0-0	=3x0+0x0+0x1-0	
			0	5/2	-3/2	3/2+m	0	0	

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New Number for Key row is

New no for key row =old element/pivot element

Soln= 2/2=1, x_1 =2/2=1, x_2 =1/2, x_3 = -1/2, x_4 =1/2, x_5 =0/2=0, x_6 =0/2=0

Other than key row new no. = old element - PCE*NPRE

Soln= 3-1*1=2, x_1 =1-1*1=0, x_2 =3-1*1/2=5/2, x_3 =0-1*-1/2=1/2, x_4 =0-1*1/2=-1/2, x_5 =1-1*0=1, x_6 =0-1*0=0

For x_6 new no are

Soln= 4-0*1=4, x_1 =0-0*1=0, x_2 =8-0*1/2=8, x_3 =0-0*-1/2=0, x_4 =0-0*1/2=0, x_5 =0-0*0=0, x_6 =1-0*0=1

			3	-1	0	-M	0	0	
Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	x_6	Min ratios
x_1	3	3	1	3	0	0	1	0	
x_3	0	4	0	5	1	-1	2	0	
x_6	0	4	0	1	0	0	0	1	
	Max Z =3x3+0x4+0x4=9		=3x1+0x0+0x0-3	=3x3+0x5+0x1-(-1)	=3x0+0x1+0x0-0	3x0+0x-1+0x0-(-M)	=3x1+0x2+0x0-0	=0x3+0x0+0x1-0	
			0	10	0	M	3	0	

New Number for Key row is

New no for key row =old element/pivot element (x_3)

Soln= 2/1/2=4, x_1 =0/1/2=0, x_2 =5/2/1/2=5, x_3 = 1/2/1/2=1, x_4 =-1/2/1/2=-1, x_5 =1/1/2=2, x_6 =0/1/2=0

Other than key row new no. x_1 row = old element – PCE *NPRE

Soln = 1-(-1/2)*4=3, x_1 =1+1/2*0=1, x_2 =1/2+1/2*5=3, x_3 =-1/2-(-1/2)*1=0, x_4 =1/2+1/2*-1=0, x_5 =0+1/2*2=1, x_6 =0-(-1/2)*0=0

For x_6 new no are

Soln= 4-0*4=4, x_1 =0-0*0=0, x_2 =1-0*5=1, x_3 =0-0*1=0, x_4 =0-0*-1=0, x_5 =0-0*0=0, x_6 =1-0*0=1

Since all the NER is +ve and at zero level which is optimum.

This problem is of case A, which means no artificial vector appears at the optimal table and therefore, the given problem as attained the optimality.

$x_1 = 3, x_2 = 0$, $x_3 = 4$, $x_4 = 0$, $x_5 = 0 x_6 = 4$ substitute this values in the objection function.

Max Z $= 3 \times 3 - 0 + 0 \times 4 - M \times 0 + 0 \times 0 + 0 \times 4 = 9$

Problem:2

Solve the given problem by charnes penalty method

Max Z $= 3x_1 + 2x_2$

STC

$2x_1 + x_2 \quad \leq 2$

$3x_1 + 4x_2 \quad \geq 12$

$\qquad x_1, \ x_2 \geq 0$

Solution:

Since, the given problem is max z

The I constraint is $\leq$ type we introduce slack variable as $x_3 \geq 0$ and the II constraint is of $\geq$ type we introduce surplus variable as $x_4 \geq 0$ and a artificial variable as $x_5 \geq 0$. In the constraint the value of artificial variable will be 1 and in the objective function is $-M$.

Therefore, the standard form of the problem

Max Z $= 3x_1 + 2x_2 + 0x_3 + 0 \ x_4 - M \ x_5$

STC

$2x_1 + x_2 + x_3 = 2$

$3x_1 + 4x_2 - x_4 + x_5 = 12$

$x_1, x_2, x_3, \ x_4 \ x_5 \geq 0$

The Matrix form

Max Z$= (3, 2, \ 0, 0, -M) \ (x_1, x_2, \ x_3, \ x_4 \ x_5)$

STC

$$\begin{Bmatrix} 2 & 1 & 1 & 0 & 0 \\ 3 & 4 & 0 & -1 & 1 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 12 \end{Bmatrix}$$

$\qquad x_1, x_2, x_3, \ x_4 \ x_5 \geq 0$

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$$\begin{Bmatrix} 1 & 0 \\ 0 & 1 \end{Bmatrix}$$

Canonical system. The basic variable (x_3, x_5) and their obvious solution
(2,12)

Starting table: 3   2   0   0   -M

Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	Min ratios
x_3	0	2	2	1	1	0	0	2/1=2
x_5	-M	12	3	4	0	-1	1	12/4=3
	Max Z=0x2+-mx12		=0x2+-mx3-3	=0x1+-mx4-2	=0x1+-mx0-0	=0 x 0 +- mx-1-0	=0x0+-m x1-(-m)	
	-12m		-3m-3	-4m-2	0	m	0	

Starting table: 3   2   0   0   -M

Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	Min ratios
x_2	2	2	2	1	1	0	0	
x_5	-m	4	-5	0	-4	-1	1	
	Maxz= 2x2+-mx4		5m+1	0	4m+2	m	0	

Since, the co-efficient of m in each $Z_j - C_j$ is non-negative and an artificial vector appeared in the basis, not at zero level.

Then, given LPP does not possess an optimum basic feasible solution. So, that is existence of pseudo optimum BFS to the given LPP.

Problem:3

Solve the given problem by charnes penalty method

Max Z $=3x_1 + 2x_2 + 3x_3$

STC

$2x_1 + x_2 + x_3 \quad \leq 2$

$3x_1 + 4x_2 + 2x_3 \quad \geq 8$

$\quad x_1, \ x_2, \ x_3 \geq 0$

Solution:

Since, the given problem is max z

The I constraint is <= type we introduce slack variable as $x_4 \geq 0$ and the II constraint is of >= type we introduce surplus variable as $x_5 \geq 0$ and a artificial variable as $x_6 \geq 0$.

In the constraint the value of artificial variable will be 1 and in the objective function is – M. Therefore, the standard form of the problem.

Max Z $=3x_1 + 2x_2 + 3x_3 + 0\,x_4 + 0\,x_5 - M\,x_6$

STC

$2x_1 + x_2 + x_3 + x_4 = 2$

$3x_1 + 4x_2 - x_5 + x_6 = 8$

$x_1, x_2, x_3, \ x_4 \ x_5 \ x_6 \geq 0$

The Matrix form

Max Z= (3,2,3, 0,0,-M) $(x_1, x_2, x_3, \ x_4 \ x_5, x_6)$

STC

$$\left\{\begin{array}{cccccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 4 & 0 & 0 & -1 & 1 \end{array}\right\} = \left\{\begin{array}{c} 2 \\ 8 \end{array}\right\} \qquad \left\{\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right\}$$

Canonical system. The basic variable

$x_1, x_2, x_3, \ x_4 \ x_5 \ x_6 \geq 0$ (x_4, x_6) and their obvious solution

(2, 8)

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Starting table:

			3	2	3	0	0	-M	
Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	x_6	Min ratios
X_4	0	2	2	1	1	1	0	0	2/1=2
x_6	-M	8	3	4	2	0	-1	1	8/4=2
			=0x2+- mx3-3	=0x1+- mx4-2	=0x1+ -mx2- 3	=0 x 1 +- mx0- 0	=0x0+ -m x- 1- 0	=0x0+- mx1-(- m)	
			-3m-3	-4m-2	-2m-3	0	m	0	

Starting table:

			3	2	0	0	0	-M	
Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	x_6	Min ratios
x_2	2	2	2	1	1	1	0	0	
x_6	-m	0	-5	0	-2	-4	-1	1	
	maxZ=2x2+- mx0=4		5m+1	1	2m+1	4m+2	m	0	

Since, the co-efficient of m in each $Z_j - C_j$ is non-negative and an artificial vector appears
in the basis, at zero level.

Then indicates the existence of an optimum basic feasible solution to the given LPP.

Thus, an optimum basic feasible solution to the given LPP.

$x_1 = 0$	$x_2 = 2$	$x_3 = 0$	$x_4 = 0$	$x_5 = 0$	$x_6 = 0$

Max Z $= 3x_1 + 2x_2 + 3x_3 + 0 x_4 + 0 x_5 - M x_6$

$\qquad = 3*0 + 2*2 + 3*0 + 0*0 + 0*0 - M*0$

$\qquad = 4$

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Problem No : 4

Solve the given problem by charnes penalty method

Max Z $=3x_1 +2x_2 + 4x_3$

STC

$2x_1 + 5x_2+ x_3 \qquad = 12$

$3x_1 +4x_2 \qquad = 11$

$x_1, x_2, x_3 \quad >= 0$

Problem No 5

Solve the given LPP

Min Z $= 4x_1 + x_2$

STC

$3x_1 + 2x_2 \qquad = \quad 3$

$4x_1 +3x_2 \quad >= \quad 6$

$x_1 +2x_2 \qquad <= \quad 4$

$x_1, \ x_2 \quad >= 0$

The I constraint is already in standard form, only artificial variable is added as $x_3 >= 0$

and II constraint is of >= type we introduce

$x_4 >= 0$ as surplus variable and an artificial variable as $x_5 >= 0$ and third constraint is of

<= type we introduce $x_6 >= 0$ as slack variable.

Then, the given LPP is in standard form

Min Z $= 4x_1 + x_2+ Mx_3+ 0x_4+ Mx_5+ 0x_6$ converting MinZ= -Z

 or

Max Z $= -4x_1 - x_2- Mx_3- 0x_4- Mx_5- 0x_6$

STC

$3x_1 + 2x_2 + x_3 \qquad = \quad 3$

$4x_1 +3x_2 -x_4+x_5 \quad = \quad 6$

$x_1 +2x_2+ x_6 \qquad = \quad 4$

$x_1,x_2,x_3,x_4,x_5,x_6 \quad >= \quad 0$

The matrix form

Max Z = (-4, -1, -M, 0, -M, 0) $(x_1, x_2, x_3, x_4, x_5, x_6)$

STC

$$\begin{Bmatrix} 3 & 2 & 1 & 0 & 0 & 0 \\ 4 & 3 & 0 & -1 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 6 \\ 4 \end{Bmatrix}$$

$x_1, x_2, x_3, x_4, x_5, x_6 >= 0$

Starting table:

			-4	-1	-m	0	-m	0	
Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	x_6	Min ratios
X_3	-m	3	3	2	1	0	0	0	3/3=1
x_5	-m	6	4	3	0	-1	1	0	6/4=1.5
x_6	0	4	1	2	0	0	0	1	4/1
	Max z= -9m		-7m+4	-5m+1	0	m	0	0	

Starting table:

			-4	-1	-m	0	-m	0	
Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	x_6	Min ratios
X_1	-4	1	1	2/3	1/3	0	0	0	3/2
x_5	-m	2	0	1/3	-4/3	-1	1	0	6
x_6	0	3	0	4/3	-1/3	0	0	1	9/4
	Max z= -4-2m		0	.-5-m /3	4m-4 /3	m	0	0	

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Starting table:			-4	-1	$-m$	0	$-m$	0	
Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	x_6	Min ratios
X_2	-1	$3/2$	$3/2$	1	$1/2$	0	0	0	
x_5	$-m$	$3/2$	$-1/2$	0	-2	-1	1	0	
x_6	0	1	-2	0	-3	0	0	1	
	Max z= $-3/2-3/2m$		$5+m/2$	0	$-1/2+2m$	m	0	0	

Problem 6

Solve the given problem by BIG M Method

Min $Z = -3x_1 + x_2 + x_3$

STC

$x_1 + -2x_2 + x_3 \qquad \le 11$

$-4x_1 + x_2 + 2x_3 \qquad \ge 3$

$24x_1 - x_3 \qquad = -1$

$\qquad x_1, x_2, x_3 \quad \ge 0$

Solution:

The I constraint is of $\le$ type so add $x_4 \ge 0$ as slack variable, II constraint is of $\ge$ type so we introduce $x_5 \ge 0$ as surplus variable and $x_6 \ge 0$ as artificial variable and III constraint is already a standard form and right side is –ve. So it should be multiplied by -1 both sides of the equation and $x_7 \ge 0$ as artificial variable and the standard form of the given LPP is written below.

Min $Z = -3x_1 + x_2 + x_3 + 0x_4 + m x_5 + x_6 + m x_7$

STC

$x_1 + -2x_2 + x_3 + x_4 \qquad = 11$

$-4x_1 + x_2 + 2x_3 + x_5 + x_6 \qquad = 3$

$-2x_1 + x_3 + x_7 \qquad = 1$

$\qquad x_1, x_2, x_3, x_4 \ x_5 x_6 x_7 \quad \ge 0$

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The matrix form

Min z = (-3,1,1,0,M,1,M) (x_1, x_2, x_3, x_4 x_5 x_6 x_7)

STC

$$
\begin{Bmatrix} 1 & -2 & 1 & 1 & 0 & 0 & 0 \\ -4 & 1 & 2 & 0 & -1 & 1 & 0 \\ -2 & 0 & 1 & 0 & 0 & 0 & 1 \end{Bmatrix}
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{Bmatrix}
= \begin{Bmatrix} 11 \\ 3 \\ 1 \end{Bmatrix}
$$

x_1, x_2, x_3, x_4 x_5 x_6 $x_7 >= 0$

Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Min ratios
	Starting table:		-3	1	1	0	m	1	m	
X_4	0	11	1	-2	1	1	0	0	0	11/1
X_6	M	3	-4	1	2	0	-1	1	0	3/2
x_7	M	1	-2	0	1	0	0	0	1	1/1
	MaxZ=0x11+3xM+ 1xm=4m		-3+6m	1-m	1-3m	0	2m	1-m	0	

NER= C_j –(P/U* x_j) (j=12….n)

x_1= -3-(0x1+mx-4+-2m) = -3+6m

x_2= 1- (0x-2+mx1+0xm) = 1-m

x_3= 1- (0x1+mx2+1xm) = 1-3m

x_4= 0- (0x1+mx0+0xm) = 0

x_5= m- (0x0+mx-1+0xm) = 2m

x_6= 1- (0x0+mx1+0xm) = 1-m

x_7= m- (0x0+mx0+1xm) = 0

First table:			-3	1	1	0	m	1	m	
Basic Variable	Profit / unit	solution	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Min ratios
X_4	0	10	3	-2	0	1	0	0	-1	10/-2= -ve
X_6	M	1	0	1	0	0	-1	1	-2	1/1=1
X_3	1	1	-2	0	1	0	0	0	1	1/0=0
	Min Z= 0x10 + mx1 +1x1 =1+m		-3-(0x3+mx0+1x-2) = -1	1-(0x-2+mx1+1x0)=1-m	1-(0x0+mx0+1x1) = 0	0-(0x1+mx0+1x0)= 0	m-(0x0+mx-1+1x0)= 2m	=1-(0x0+mx1+1x0) = 1-m	m -(0x-1+mx-2+1x1) = 3m-1	

The values of key row remain the same because 1 is the key element

New nos for other than key rows i.e X_4

Soln= 11-1*1=10 X_5,Soln= 3-2*1=1

x_1= 1-1*-2 =3 x_1= -4-2*-2= 0

x_2= -2-1*0=-2 x_2= 1-2*0=1

x_3=1-1*1=0 x_3= 2-2*1=0

x_4=1-1*0=1 x_4= 0 -2*0=0

x_5=0-1*0=0 x_5= -1-2*0=-1

x_6=0-1*0=0 x_6= 1-2*0=1

x_7=0-1*1=-1 x_7=0-2*1=-2

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Third table:

Basic Variable	Profit / unit	Solution	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Min ratios
			-3	1	1	0	m	1	m	
X_4	0	12	3	0	0	1	2	-2	-5	12/3
X_2	1	1	0	1	0	0	1	-1	-2	1/0
X_3	1	1	-2	0	1	0	0	0	1	1/-2
			-1	0	0	0	m-1	1	m+1	

Fourth table:

Basic Variable	Profit / unit	Solution	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Min ratios
			-3	1	1	0	m	1	m	
X_1	-3	4	1	0	0	1/3	2/3	-2/3	-5/3	
X_2	1	1	0	1	0	0	1	-1	-2	
X_3	1	9	0	0	1	2/3	4/3	-4/5	-7/3	
			0	0	0	1/3	m-1/3	1/3	m-2/3	

Table 4 is optimal and the value of the decision variable is

$x_1 = 4$, $x_2 = 1$, $x_3 = 9$, $x_4 = 0$, $x_5 = 0$, $x_6 = 0$ $x_7 = 0$

Min Z = -3*4 +1*1 + 1*9 + 0*0 +0*0+ 0*0 +0*)

$\qquad$ =-12+1+9

$\qquad$ = -2

Duality Theory

The linear programming model we develop for a situation is referred to as the **primal** problem. The dual problem can be derived directly from the primal problem.

The standard form has three properties

1. All the constraints are equations (with nonnegative right-hand side)
2. All the variables are nonnegative
3. The sense of optimization may be maximization of minimization.

Comparing the primal and the dual problems, we observe the following relationships.

1. The objective function coefficients of the primal problem have become the right-hand side constants of the dual. Similarly, the right-hand side constants of the primal have become the cost coefficients of the dual
2. The inequalities have been reversed in the constraints
3. The objective is changed from maximization in primal to minimization in dual
4. Each column in the primal correspondence to a constraint (row) in the dual. Thus, the number of dual constraints is equal to the number of primal variables.
5. Each constraint (row) in the primal corresponds to a column in the dual. Hence, there is one dual variable for every primal constraints
6. The dual of the dual is the primal problem.
7. if the primal constraints $>=$ the dual constraints will be $<=$ & vice versa

Duality is an extremely important and interesting feature of linear programming. The various useful aspects of this property are

i) If the primal problem contains a large no of rows constraints and smaller no of columns variables computational procedure can be considerably reduced by converting it into dual and then solving it.

ii) It gives additional information as to how the optimal solution changes as a result of the changes in the coefficients and the formulation of its problem.

iii) Calculations of the dual checks the accuracy of the primal solution

iv) This indicates that fairly close relationships exist between LP and theory of games

Primal Problem	Dual Problem
Max Z = CX STC Ax<=b x>=0	Min Z = bY STC Ay>=c y>=0
Max Z = CX STC ex: Ax=b x1+x2<=2 x>=0 -x1-x2<=-2	Min Z = bY STC Ay>=c Y is unrestricted in sign
Max Z = CX STC Ax=b X is unrestricted in sign	Min Z = bY STC Ay=c y is unrestricted in sign
Max Z = CX STC A1x<=b1 A2x=b2 x>=0	Min Z = b1Y1+b2y2 STC A1y1+A2y2>=c Y1>=0 y2 is unrestricted in sign

Note: it is not necessary that only the Max problem be taken as the primal problem we can as well consider the minimization LPP as the primal

Formulation of dual to the primal problem

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Problem no 1

Write the dual of the primal problem

Max $Z = 3x_1 + 5x_2$

STC

$2x_1 + 6x_2 \qquad \leq 50$

$3x_1 + 2x_2 \qquad \leq 35$

$5x_1 - 3x_2 \qquad \leq 10$

$\qquad x_2 \qquad \leq 20$

$\quad x_1, x_2, x_3 \qquad \geq 0$

To write dual for the above primal, since the primal as 4 constraints the dual will have 4 variables as y_1 y_2 y_3 y_4 then dual for the primal will be as follows.

Min $Z = 50y_1 + 35y_2 + 10y_3 + 20y_4$

STC

$2 y_1 + 3y_2 + 5y_3 + 0y_4 \geq 3$

$6y_1 + 2y_2 - 3y_3 + y_4 \quad \geq 5$

$\qquad y_1, y_2, y_3, y_4 \quad \geq 0$

it can be observed from the dual problem has less no constraint as compared to the primal problem (in case of primal they are 4 and in case of dual they are 2) which requires less work and effort to solve it

Problem 2

Construct the dual of the given problem

Min $Z = 3x_1 - 2x_2 + 4x_3$

STC

$3x_1 + 5x_2 + 4 x_3 \qquad \geq 7$

$6x_1 + x_2 + 3x_3 \qquad \geq 4$

$7x_1 - 2 x_3 - x_3 \qquad \leq 10$

$x_1 - 2x_2 + 5 x_3 \qquad \geq 3$

$4x_1 + 7x_2 - 2 x_3 \qquad \geq 2$

$\quad x_1, x_2, x_3 \qquad \geq 0$

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Solution:

The given problem is minimization type and all the constraint should be >= type. In the given problem third constraint is <= type so we must convert constraint to >= type by multiply both side of the constraint by -1, we get

$-7x_1 + 2x_3 + x_3 \quad >= -10$

Then given the problem can be written restated

Min $Z = 3x_1 - 2x_2 + 4x_3$

STC

$3x_1 + 5x_2 + 4x_3 \quad >= 7$

$6x_1 + x_2 + 3x_3 \quad >= 4$

$-7x_1 + 2x_2 + x_3 \quad >= -10$

$x_1 - 2x_2 + 5x_3 \quad >= 3$

$4x_1 + 7x_2 - 2x_3 \quad >= 2$

$x_1, x_2, x_3 \quad >= 0$

Then dual of the given problem is as follows and the dual variables are y_1, y_2, y_3, y_4, y_4

Max $Z = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$

STC

$3y_1 + 6y_2 - 7y_3 + y_4 + 4y_4 \quad >= 3$

$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_4 \quad >= -2$

$4y_1 + 3y_2 + y_3 + 5y_4 - 2y_4 \quad >= 4$

$y_1, y_2, y_3, y_4, y_5 \quad >= 0$

Problem 3

Obtain the dual problem of the following LPP

Max $Z = 2x_1 + 5x_2 + 6x_3$

STC

$5x_1 + 6x_2 - 4x_3 \quad\quad \leq 3$

$-2x_1 + x_2 + 4x_3 \quad\quad \leq 4$

$x_1 - 5x_2 + 3x_3 \quad\quad \leq 1$

$-3x_1 - 3x_2 + 7x_3 \quad\quad \leq 6$

$x_1, x_2, x_3 \quad\quad \geq 0$

Also, verify that the dual of the dual problem is the primal problem

The given primal can be restated in the matrix form

Max $Z = (2,5,6)(x_1, x_2, x_3)$

STC

$$\begin{Bmatrix} 5 & 6 & -4 \\ -2 & 1 & 4 \\ 1 & -5 & 3 \\ -3 & -3 & 7 \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \leq \begin{Bmatrix} 3 \\ 4 \\ 1 \\ 6 \end{Bmatrix}$$

$$x_1, x_2, x_3 \quad\quad \geq 0$$

The dual of the above primal can be written as and the dual variables are (y_1, y_2, y_3, y_4)

Min $w = (y_1, y_2, y_3, y_4)(3.4.1.6)$

STC

$$\begin{Bmatrix} 5 & -2 & 1 & -3 \\ 6 & 1 & -5 & 3 \\ -4 & 4 & 3 & 7 \end{Bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix} \geq \begin{Bmatrix} 2 \\ 5 \\ 6 \end{Bmatrix}$$

$$y_1, y_2, y_3, y_4 \geq 0$$

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Now, we can restate this dual problem as follows

Max w = (-3.-4.-1.-6) (y_1, y_2, y_3, y_4)

$$(-1)\begin{Bmatrix} 5 & -2 & 1 & -3 \\ 6 & 1 & -5 & -3 \\ -4 & 4 & 3 & -7 \end{Bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix} = \begin{Bmatrix} -2 \\ -5 \\ -6 \end{Bmatrix}$$

$y_1, y_2, y_3, y_4 >= 0$

$MaxZ = (-1) Minz$

The dual problem in this form looks like the primal problem and this we may write down

the dual of this dual problem .

MinZ= (-2,-5, -6) (x_1, x_2, x_3)

STC

$$(-1)\begin{Bmatrix} 5 & 6 & -4 \\ -2 & 1 & 4 \\ 1 & -5 & 3 \\ -3 & -3 & 7 \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} <= \begin{Bmatrix} -3 \\ -4 \\ -1 \\ -6 \end{Bmatrix}$$

$x_1, x_2, x_3 >= 0$

or Max z = (2 5 6) (x_1, x_2, x_3)

STC

$$\begin{Bmatrix} 5 & 6 & -4 \\ -2 & 1 & 4 \\ 1 & -5 & 3 \\ -3 & -3 & 7 \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} <= \begin{Bmatrix} -3 \\ -4 \\ -1 \\ -6 \end{Bmatrix}$$

$x_1, x_2, x_3 >= 0$

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Problem 4

Construct the dual of the primal problem

Max $Z = 3x_1 + 17x_2 + 4x_3$

STC

$x_1 - x_2 + x_3 \geq 3$

$-3x_1 + 2x_3 \leq 1$

$x_1, x_2, x_3 \geq 0$

Problem 5

Construct the dual of the following primal

Max $Z = x_1 - 2x_2 + 3x_3$

STC

$-2x_1 + x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$

$x_1, x_2, x_3 \geq 0$

The given problem is 2nd variety in the duality and it is in the form of Max Z, then the constraint in the standard form can be written as

$-2x_1 + x_2 + 3x_3 \leq 2$

$2x_1 - x_2 - 3x_3 \leq -2$

and

$2x_1 + 3x_2 + 4x_3 \leq 1$

$-2x_1 - 3x_2 - 4x_3 \leq -1$

Then, given primal can restated as

Max $Z = x_1 - 2x_2 + 3x_3$

STC

$-2x_1 + x_2 + 3x_3 \leq 2$

$2x_1 - x_2 - 3x_3 \leq -2$

$2x_1 + 3x_2 + 4x_3 \leq 1$

$-2x_1 - 3x_2 - 4x_3 \leq -1$

$x_1, x_2, x_3 \geq 0$

The dual of the above primal is written as and its dual variables are y_1, y_2, y_3

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Min W = $2y_1 - 2y_2 + y_3 - y_4$
STC

$-2y_1 + 2y_2 + 2y_3 - 2y_4 \geq 1$

$y_1 - y_2 + 3y_3 - 3y_4 \geq -2$

$3y_1 - 3y_2 + 4y_3 - 4y_4 \geq 3$

$y_1, y_2, y_3, y_4 \geq 0$

This can be written

 Min W = $2y_1 + y_2$

STC

$-2y_1 + 2y_2 \geq 1$

$y_1 + 3y_2 \geq -2$

$3y_1 + 4y_2 \geq 3$

y_1, y_2 are unrestricted in sign

Problem 6

Write the dual of the given LPP

Max Z = $3x_1 + x_2 + 2x_3 - 2x_4$

STC

$2x_1 - x_2 + 3x_3 + x_4 = 1$

$x_1 + x_2 - x_3 + x_4 = -3$

$x_1, x_2 \geq 0$, x_3 & x_4 are unrestricted in sign

Problem 7

Write the dual of the following LPP

Min Z = $2x_1 + 3x_2 + 4x_3$

STC

$2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 = 2$

$x_1 + 4x_2 + x_3 \leq 5$

$x_1, x_2, \geq 0$ x_3 are is unrestricted in sign
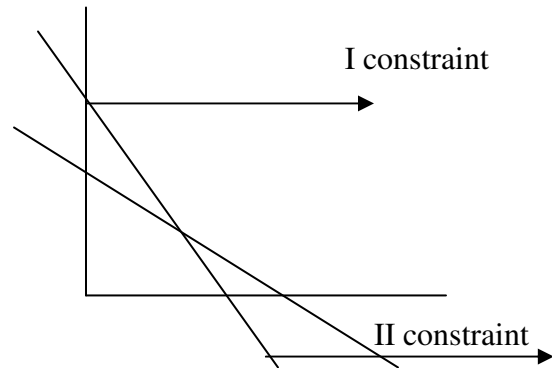
Use duality to solve the following LPP

Max $Z = 3x_1 + 2x_2$

STC

$2x_1 + x_2 \le 5$ the points are (5/2,5)

$x_1 + x_2 \le 3$ (3,3)

$x_1, x_2 \ge 0$

I constraint

II constraint

Use duality concept the dual of the above primal as follows

Min $Z = 5y_1 + 3y_2$

STC

$2y_1 + y_2 \ge 3$ (3/2,3)

$y_1 + y_2 \ge 2$ (2,2)

$y_1, y_2 \ge 0$

I constraint

II constraint
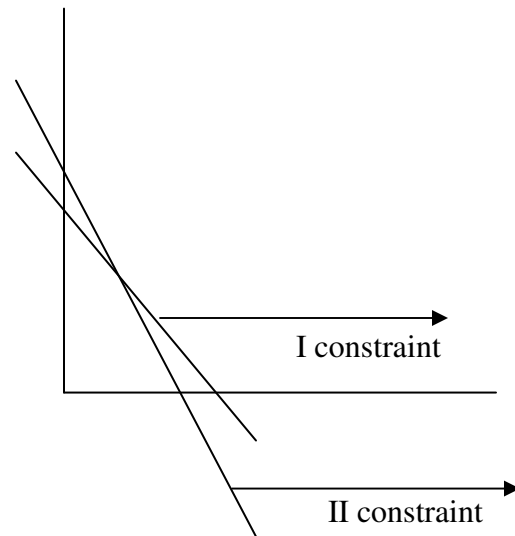
Using duality theory solve the following LPP by simplex method

Min $Z = 4x_1 + 3x_2 + 6x_3$

STC

$x_1 + x_3 \ge 2$

$x_2 + x_3 \ge 5$

$x_1, x_2, x_3 \ge 0$

the dual of the above primal

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Max $Z = 2y_1 + 5y_2$

STC

$y_1 <= 4$

$y_2 <= 3$

$y_1 + y_2 <= 6$

$y_1, y_2 >= 0$

Using duality theory solve the following LPP by simplex method

Max $Z = x_1 + 6x_2$

STC

$x_1 + x_2 \quad >= 2$

$x_1 + x_2 \quad <= 3$

$x_1, x_2 \quad >= 0$

The given problem is max Z so all the constraint should be <= type. In this problem I constraint is of >= type so we converting by multiply b -1 both sides of the constraint the resulting will be as

$-x_1 - x_2 \quad <= -2$

Then, the given LPP is restated as

Max $Z = x_1 + 6x_2$

STC

$-x_1 - x_2 \quad <= -2$

$x_1 + x_2 \quad <= 3$

$x_1, x_2 \quad >= 0$

The dual of the above LPP will be as

Min $Z = 2y_1 + 3y_2$

STC

$-y_1 + y_2 \quad >= 1$

$-y_1 + y_2 \quad >= 6$

$x_1, x_2 \quad >= 0$

Solving the dual using Big M method

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Dual simplex method

Dual simplex method applies to problems which start with dual feasible solns. The objective function may be either in the maximization form or in the minimization form. After introducing the slack variables, if any right-hand side element is –ve and if the optimality condition is satisfied.

The problem can be solved by the dual simplex method

Procedure for dual simplex method

Step1) obtain an initial basic solution to the LPP and put the solution in the starting dual simplex table

Step2) test the nature of Z_j-C_j in the starting simplex table

a)if all Z_j-C_j and solution column are non-negative for all I and j, then an optimum basic feasible solution has been obtained.

b) if all Z_j-C_j are non negative and at least on basic variable in the solution column is negative go step 3

c) if at least one Z_j-C_j is –ve the method is not applicable to the given problem

Step 3) selects the most –ve in solution column

step4) test the nature of

> a) if all x_{ij} are non negative the given problem does not exist any feasible solution
>
> b) b) if at least one x_{ij} is –ve, compute the ratios Z_j-C_j / x_{ij} , $x_{ij} >= 0$ chose the maximum of theses ratios

Step 5) test the new iterated dual simplex table for optimality

Repeat the procedure until either an optimum feasible solution has been obtained or there is an indication of the non existence of a feasible solution.

Use dual simplex method to solve the LPP

Min $Z = x_1 + x_2$

STC

$2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

Since, the given problem in min z form it is converted to max z and all the constraint is of $\geq$ type it should be converted to $\leq$ type by multiplying -1 on the both sides then given problem will be as follows

Max $Z = -x_1 - x_2$

STC

$-2x_1 - x_2 \leq -2$

$x_1 + x_2 \leq -1$

$x_1, x_2 \geq 0$

Now, introducing slack variable for the I and II constraint as $x_3 \geq 0, x_4 \geq 0$

The standard form as follows

Max $Z = -x_1 - x_2 + 0 x_3 + 0x_4$

STC

$-2x_1 - x_2 + x_3 = -2$

$x_1 + x_2 + x_4 = -1$

$x_1, x_2, x_3 x_4 \geq 0$

The matrix form

Max $Z = (-1, -1, 0, 0) (x_1, x_2, x_3 x_4)$

STC

$$\begin{bmatrix} -2 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

the identity matrix form are

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ the correspond variables

$x_1, x_2, x_3 x_4 \geq 0$

$(x_3 x_4)$ and

their solutions are -2 -1

starting table			-1	-1	0	0
Basic Variable	Profit / Unit	solution	x_1	x_2	x_3	x_4
x_3	0	-2	-2	-1	1	0
x_4	0	-1	1	1	0	1
	Max Z= 0*-2+0*-1		=0*-2+0*1+1	=0*-1+0*1+1	=0*1+0*0 -0	=0*0+0*1-0
	0		1	1	0	0

Ratios =   NER / X_{1j}

$$= \begin{bmatrix} 1/-2, \ 1/-1, \ 0/1, \ 0/0 \end{bmatrix}$$

The max negative in the ratios are 1/-2

First table			-1	-1	0	0
Basic Variable	Profit / Unit	solution	x_1	x_2	x_3	x_4
x_1	-1	1	1	0.5	-1/2	0
x_4	0	-2	0	0.5	1/2	1
	Max Z= -1*1=0*-2		=-1*1+0*0+1	=0.5*-1+0.5*0+1	=-1*-1/2+0*1/2 -0	=-1 * 0 + 0*1-0
	-1		0	0.5	0	0

New no for key row are

Soln = -2/-2 = 1, -2/-2 =1, -1/-2=0.5, - 1/2,0/-2

New no for other than key rows is x_4

Soln=-1-1*1=-2, 1-1*1=0, 1-1*0.5=0.5, 0-1*-1/2=1/2, 1-1*0=1

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Since, all the values in NER is positive and in solution column one variable is –ve and that is selected as key row and to select key column or pivot column at least on variable in the row should be –ve but, no vector corresponding to that row is –ve and we cannot find the ratios and So we cannot select the key column and the given problem does not given any feasible solution to the LPP.

Use dual simplex table to solve the LPP

Min $Z = x_1 + 2x_2 + 3x_3$

STC

$x_1 - x_2 + x_3 \geq 4$

$x_1 + x_2 + 2x_3 \leq 8$

$x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

the given problem is set for the requirement of dual simplex method

Max $Z = -x_1 - 2x_2 - 3x_3$

STC

$-x_1 + x_2 - x_3 \leq -4$

$x_1 + x_2 + 2x_3 \leq 8$

$-x_2 + x_3 \leq -2$

$x_1, x_2, x_3 \geq 0$

since, all the constraint are of <= type we introduce three slack variable for I II and III constraint as $x_4 \geq 0$, $x_5 \geq 0$, $x_6 \geq 0$

the standard form

Max $Z = -x_1 - 2x_2 - 3x_3 + 0x_4 + 0x_5 + 0x_6$

STC

$-x_1 + x_2 - x_3 + x_4 = -4$

$x_1 + x_2 + 2x_3 + x_5 = 8$

$-x_2 + x_3 + x_6 = -2$

$x_1, x_2, x_3 x_4 x_5 x_6 \geq 0$

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The matrix form

Max Z $(-1,-2,-3,0,0,0)$ $(x_1, x_2, x_3\ x_4\ x_5\ x_6)$

STC

$$
\begin{bmatrix}
-1 & 1 & -1 & 1 & 0 & 0 \\
1 & 1 & 2 & 0 & 1 & 0 \\
0 & -2 & 1 & 0 & 0 & 1
\end{bmatrix}
\begin{Bmatrix}
x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6
\end{Bmatrix}
=
\begin{Bmatrix}
-4 \\ 8 \\ -2
\end{Bmatrix}
\begin{Bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{Bmatrix}
$$

The IBFS are $x_4\ x_5\ x_6$

$x_1, x_2, x_3\ x_4\ x_5\ x_6 >= 0$ and their obvious soln (-4,8,-2)

Starting table   -1   -2   -3   0   0   0

Basic Variable	Profit / unit	soln	x_1	x_2	x_3	x_4	x_5	x_6
x_4	0	-4	-1	1	-1	1	0	0
x_5	0	8	1	1	2	0	1	0
x_6	0	-2	0	-1	1	0	0	1
	MaxZ =0*-4+0*8+0*-8		1	2	3	0	0	0

The ratios =(1/-1, 2/-1, 3/-1, 0/0, 0/0, 0/1)

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Other than key row new nos are

x_5

soln = 8-1*4=4, x_1=1-1*1=0, x_2=1-1*-1=0, x_3=2-1*1=1, x_4=0-1*-1=1

x_5 = 1-1*0=1, x_6=0-1*0=0

x_6

soln= -2-0*4=-2, all other value in the row remains the same because of corresponding

value of pivot column element is zero

Starting table			-1	-2	-3	0	0	0
Basic Variable	Profit / unit	soln	x_1	x_2	x_3	x_4	x_5	x_6
x_1	-1	4	1	-1	1	-1	0	0
x_5	0	4	0	0	1	1	1	0
x_6	0	-2	0	-1	1	0	0	1
	Max Z =		0	3	2	1	0	0

Ratios = (0/0, 3/-1,2/1,1/0,0/0,0/1)

For x_1 row

soln=4-(-1)*2=6, x_1=1-(-1)*0=1, x_2=-1-(-1)1=0, x_3=1-(-1)-1=0, x_4=-1-(-1)*0=-1, x_5=0-(-1)*0=0, x_6=0-(-1)*-1=-1

For x_5 row

soln =4 - 0*2=4, any thing multiplied by 0 is 0 so the value in this row remains same.

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Starting table   -1   -2   -3   0   0   0

Basic Variable	Profit / unit	soln	x_1	x_2	x_3	x_4	x_5	x_6
x_1	-1	6	1	0	0	-1	0	-1
x_5	0	4	0	0	1	1	1	0
x_2	-2	2	0	1	-1	0	0	-1
	Max Z =		0	0	5	1	0	3

Since all the NER and Solution column are non negative and the   given problem as attained the optimum

Therefore, x_1 =6, x_2 =2, x_3 =0, x_4 =0, x_5 =4, x_6 =0

These values are substituted in the objective function

Max Z = -6*1 -2*2 -3*0 +0*0+ 0*4+ 0*0

Dual Read off Technique

Solve the following the LPP and read the solution for the real problem

Min $Z = 2x_1 + x_2$

STC

$x_1 + x_2$ $>= 15$

$x_1 - x_2$ $<= 3$

$2x_1 + 5x_2$ $>= 20$

$-x_1 + 3x_2$ $>= 10$

x_1, x_2 $>= 0$

To write the dual of the above primal all the constraint should be of $>=$ type, since, II constraint is $<=$ type we must convert it into $>=$ type by -1, then, the given primal can be written as follows

Min $Z = 2x_1 + x_2$

STC

$x_1 + x_2$ $>= 15$

$-x_1 + x_2$ $>= -3$

$2x_1 + 5x_2$ $>= 20$

$-x_1 + 3x_2$ $>= 10$

x_1, x_2 $>= 0$

The dual for the above primal problem as follows

The dual variables (y_1 y_2 y_3 y_4)

Max $Z = 15y_1 - 3y_2 + 20y_3 + 10y_4$

STC

$y_1 - y_2 + 2y_3 - y_4$ $<= 2$ $y_5 >= 0$ as slack variable

$y_1 + y_2 + 5y_3 + 3y_4 <= 1$ $y_6 >= 0$ as slack variable

y_1 y_2 y_3 $y_4 >= 0$

Matrix form

Max Z = $(y_1 \ y_2 \ y_3 \ y_4)$ (15,-3,20,10)

STC

$$\begin{Bmatrix} 1 & -1 & 2 & -1 & 1 & 0 \\ 1 & 1 & 5 & 3 & 0 & 1 \end{Bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$$

$y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 >= 0$

$\begin{Bmatrix} 1 & 0 \\ 0 & 1 \end{Bmatrix}$ is identity matrix which form the basic variable ($y_5 \ y_6$) for the I table and it's solution is (2,1)

Starting table   15   -3   20   10   0   0

Basic Variable	Profit / unit	soln	y_1	y_2	y_3	y_4	y_5	y_6	Min ratio
y_5	0	2	1	-1	2	-1	1	0	2/2=1
y_6	0	1	1	1	5	3	0	1	1/5=
	Max Z = 0		-15	3	-20	-10	0	0	

Starting table   15   -3   20   10   0   0

Basic Variable	Profit / unit	soln	y_1	y_2	y_3	y_4	y_5	y_6	Min ratio
y_5	0	8/5	3/5	0	-10/5	1	-1	-2/5	8/3
y_3	20	1/5	1/5	1/5	1	3/5	0	1/5	1
	Max Z = 0		-11	7	0	2	0	20	

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Starting table   15   -3   20   10   0   0

Basic Variable	Profit / unit	soln	y_1	y_2	y_3	y_4	y_5	y_6	Min ratio
y_5	0	∞	0	-2	-3/5	-4	1	∞	
y_1	15	1	1	1	5	3	0	1	
	Max Z = 0		0	18	55	35	0	15	

Surplus variable   Real Primal

Since all the NER non negative the given problem as attained optimum

Sl. No.	Primal	Dual
1	$X_1 = 0$	$Y_1 = 1$
2	$X_2 = 15$	$Y_2 = 0$
3	$X_3 = 0$	$Y_3 = 0$
4	$X_4 = 18$	$Y_4 = 0$
5	$X_5 = 55$	$y_5 = \infty$
6	$x_6 = 35$	$Y_6 = 0$

Reference Books:

1. **Taha H A**, Operation Research - An Introduction, Prentice Hall of India, 7th edition, 2003
2. **Ravindran, Phillips and Solberg**, Operations Research : Principles and Practice, John Wiely & Sons, 2nd Edition
3. **D.S.Hira**, Operation Research, S.Chand & Company Ltd., New Delhi, 2004
4.