II PUC Mock Paper
Mathematics

Duration: 3.15 minutes **Max.Marks: 100**

Section – A

I. Answer all the following questions: **10 × 1 = 10**

1. Let * be an operation defined on the set Q of rational numbers by a*b = $\frac{ab}{4}$. Find the identity element.
2. Find the value of $\sin^{-1}(\sin \frac{2\pi}{3})$.
3. Construct 2 ×2 matrix A = [a_{ij}] whose elements are given by $\frac{1}{2}\left|-3i+j\right|$.
4. Find the value of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.
5. Find the derivative of $\log_7 (\cos^x)$ w.r.t s.
6. Find the anti derivative of $e^x \left(\frac{x-1}{x^2}\right)$dx.
7. Define negative of a vector.
8. Find the intercepts cut off by the plane 2x + y – z = 5.
9. Define optimal solution of LPP.
10. If P(A) = $\frac{3}{5}$, P (B) = $\frac{1}{5}$, find P (A∩B) if A and B are independent events.

Section – B

II. Answer any ten of the following: **10 × 2 = 20**

11. Prove that if f : A → B and g:B→C are bijective then gof : A →C is also bijective.
12. Write the simplest from of $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$
13. Find $\tan\left[\frac{1}{2}\left(\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right)\right], |x|1$, y > 0 and xy < 1.
14. Find the values of k if area of triangle is 4 sq. units and vertices are (-2,0), (0,4) (0,k).
15. Differentiate $\frac{\sqrt{(x^2-5x+8)\ (x^2+7x+9)}}{x+3}$ w.r.t x by logarithmic differentiation.
16. Find the slope of the tangent to the curve $y=\frac{x-1}{x-2}, x \neq 2$ at x = 10.
17. Using differentials, find the approximate value of $\sqrt{49.05}$.
18. Find $\int \frac{1}{\sin x \cos^3 x}dx.$
19. Evaluate $\int \frac{1}{e^x+e^{-x}}dx.$
20. Find the order and degree, if the differential equation is $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0.$

21. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$

22. Find the equation of the plane through the intersection of the planes 3x – y + 2z – 4 =0 and x+y –z - 2 =0 and the point (2, 2 1).

23. If $\propto, \beta, \gamma$ are the angles made by a vector with the coordinate axes. Show that cos $2\propto + \cos\beta + \cos 2\gamma = 1$.

24. Find the probability distribution of the number of tails in simultaneous tosses of three coins.

Section – C

III. Answer any ten of the following: 10 × 3 = 30

25. Show that the relation R in the set Z of integers given by R = {(a, b) : 2 divides a-b} is an equivalence relation.

26. Find the value of x, if $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

27. Using elementary transformation, find the inverse of the matrix $A=\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}$.

28. Verify mean value theorem if f(x) = x^3 -5x^2 – 3x, in the interval [1, 3]. Find all $C\in(1,3)$ for which f `(c)=0

29. If x = a$\cos^3\theta$ and y = a $\sin^3\theta$, prove that $\frac{dy}{dx}=-\sqrt[3]{\frac{y}{x}}$

30. Find local maximum and local minimum values of the function f given by f (x) = $3x^4 + 4x^3 - 12x^2 + 12$.

31. Evaluate $\int\left(\frac{1+\log x}{x}\right)^2 dx.$

32. Integrate $\frac{2x}{(x^2+1)(x^2+2)}$ with respect to x.

33. Find the area bounded by the parabola $y^2 = 4x$ and the line y = 2x.

34. Find the distance between the lines l_1 and l_2 given by $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$.

35. Show that the position vector of the point, which divides the line joining the points A and B having the position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio m : n is $\frac{m\vec{b}+n\vec{a}}{m+n}$.

36. Form the differential equation of the family of circles touching the y-axis at origin.

37. Find the cartesian and vector equation of the line that passes through the points (3, -2, -5) and (3,-2,6)

38. Box – I contains 2 gold coins, while another Box-II contains 1 gold and 1 silver coin A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

Section – D

IV. Answer any six of the following: 5 × 6 = 30

39. Let f N →R defined by f(x) = $4x^2$ +12x + 15, prove that f : N →S, where S is the range of the function, is invertible. Also find the inverse of f.

40. If $A=\begin{bmatrix}2 & 0 & 1\\ 0 & -3 & 0\\ 0 & 0 & 4\end{bmatrix}$, verify $A^3 - 3A^2 - 10A + 24I = 0$, where O is zero matrix of order 3 ×3.

41. Solve the system of linear equation by matrix method: 2x – 3y+5z = 11; 3x +2y-4y = - 5; x+y-2z = -3.

42. If y = $(\sin^{-1} x)^2$. Show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2.$

43. Sand is pouring from a pipe at the rate of $12cm^3$/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one – sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm?

44. Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$

45. Find the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ w.r.t x and hence evaluate $\int \frac{dx}{\sqrt{7-6x-x^2}}$.

46. Find the general solution of the differential equation e^x. tan y dx + $(1-e^x)$. $\sec^2 y$. dy = 0.

47. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and cartesian form.

48. A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$.

What is the probability that he will win a prize atleast once and exactly once.

Section – E

V. Answer any one of the following: 1 × 10 = 10

49. a) Prove that $\int_{-a}^{a} f(x).dx = \begin{cases} 2\int_{0}^{a} f(x)dx, & \text{if f(x) is an even function} \\ 0 \quad , & \text{if f (x) is an odd function} \end{cases}$

and evaluate $\int_{-1}^{1} Sin^5 x \cos^4 x dx.$

b) Prove that $\begin{bmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{bmatrix} = 2(x+y+z)^3$

50. a) Minimize and maximize z = 5x +10y subject to the constrains x+2y ≤ 120; x +y ≥60 ; x – 2y ≥ 0 and x ≥ 0, y ≥ 0 by graphical method.

b) Determine the value of k, if f (x) = $\begin{cases} x^2 \sin\frac{1}{x} & ,if \quad x \neq 0 \\ k & ,if \quad x = 0 \end{cases}$

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