

PART A

I. Answer all the questions:

1 × 10 = 10

- Let * be a binary operation defined on the set of rational numbers defined by $a*b = 7ab + 7$. Verify whether * is a binary operation.
- Find the principal value of $\sec^{-1}(-\sqrt{2})$
- Define identity matrix.
- Find K if the matrix $\begin{bmatrix} K & 4 \\ 3 & 2 \end{bmatrix}$ has no inverse.
- Find the derivative of $\tan x^\circ$ {x degree}
- Find the anti derivative of $(2\cos^2 3x - 1)dx$
- Find the value of λ for the vector $\vec{a} = 2\hat{i} - 3\lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ are perpendicular to each other.
- What is the equation of the plane that cuts the co-ordinate axes at (a, 0, 0) (0, b, 0) and (0, 0, c).
- Define objective function of LPP.
- If $P(A \cap B) = 0.32$ and $P(A) = 0.8$. Find $P(B/A)$

PART B

II. Answer any 10 questions:

10 × 2 = 20

- If $f : R \rightarrow R$ given by $f(x) = (3 - x^3)^{1/3}$. Then find (fof) (x).
- Write the simple form of $\tan^{-1} \left[\frac{\sqrt{1 + \cos 2x}}{\sqrt{1 - \cos 2x}} \right]$
- If $\operatorname{cosec} \left[\tan^{-1} \left(\frac{1}{7} \right) + \cot^{-1} x \right] = 1$, find x
- Find the area of the triangle whose vertices are (3, 8), (-4, 2) and (5, 1) using determinants.
- If $x = \cos^{-1}(2t^2 - 1)$ and $y = \sin^{-1}(4t^3 - 3t)$ prove that $\frac{dy}{dx} = -\frac{3}{2}$
- Prove that $f(x) = x^3 - 3x^2 + 4x$, $x \in R$ is strictly increasing on R.
- Find the points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to x-axis.
- Evaluate $\int \frac{\cot(1 + \log x)}{x} dx$
- $\int x \sin^3(x^2) \cos x^2 dx$
- Find the order and degree of $\left[\frac{d^3 y}{dx^3} + y^2 \right]^{3/2} = e^{y^{111}}$
- Prove that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$
- Find the shortest distance between $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$
- If α, β, γ are the angles made by a vector with co-ordinate axes then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

24. Two dice are thrown. Find the probability of getting an odd number on the first die and a multiple of 3 on the other.

PART C

III. Answer any 10 questions:

10 × 3 = 30

25. Verify whether $f(x) = \frac{x-2}{x-3}$ is one-one and onto or not give reason.

26. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

27. Express $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as sum of symmetric and skew symmetric matrix.

28. If $\sqrt{1+x} + \sqrt{1+y} = 0$ P.T $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

29. If $x = a^{\sqrt{\tan^{-1}t}}$ $y = a^{\sqrt{\cot^{-1}t}}$ P.T $\frac{dy}{dx} = \frac{-y}{x}$

30. Verify mean value theorem for the function $f(x) = x^2 - 4x - 3$ in the interval $[1, 4]$

31. Evaluate $\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$

32. Find $\int \frac{x}{(x+1)(x+2)} dx$

33. Find the area of the region bounded by $y^2 = 9x$, $x = 2$ and $x = 4$.

34. Prove that $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ is a homogeneous differential equation.

35. Find a vector perpendicular to each of the vectors $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ which has magnitude 10 units.

36. If $\vec{a} = -4\hat{i} - 6\hat{j} - \lambda\hat{k}$

$$\vec{b} = -\hat{i} + 4\hat{j} + 3\hat{k}$$

$$\vec{c} = -8\hat{i} - \hat{j} + 3\hat{k} \text{ are coplanar. Find } \lambda.$$

37. Find the distance between parallel lines $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + m(2\hat{i} + 3\hat{j} + 6\hat{k})$ and

$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + n(2\hat{i} + 3\hat{j} + 6\hat{k})$$

38. Given that 2 numbers appearing on throwing two dice are different. Find the probability of the event "the sum of numbers on the dice is 4".

PART D

IV. Answer any 6 questions:

6 × 5 = 30

39. Let R^+ be the set of all non-negative real numbers. Prove that $f : R^+ \rightarrow [4, \infty]$ defined by $f(x) = x^2 + 4$ is invertible. Also write the inverse of f .

40. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$ then prove that $A(BC) = (AB)C$

41. Using matrix method solve the following equations

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

42. If $y = 5\cos(\log x) + 7\sin(\log x)$. Prove that $x^2 y_2 + xy_1 = 0$

43. A man of height 2 meters walks at a uniform speed of 5 km/hr away from a lamp post which is 6 meters height. Find the rate at which the length of his shadow increases.
44. Find the integral of $\frac{1}{\sqrt{x^2 + a^2}}$ w.r.t x and hence evaluate $\int \frac{1}{\sqrt{x^2 + 2x + 4}} dx$
45. Find the area of the region enclosed by the parabola $x^2 = 4y$ and the line $x = 4y - 2$ and x-axis.
46. Derive the equation of the line in space passing through a point and parallel to a vector both in the vector and Cartesian form.
47. Solve the differential equation $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$
48. If a fair coin is tossed 8 times. Find the probability of (i) at least five heads and (ii) at most five heads.

PART E

V. Answer any 1 question:

1 × 10 = 10

49. a) Prove $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$ then prove

$$\begin{cases} \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx & \text{when } f(2a - x) = f(x) \\ \int_0^{2a} f(x) dx = 0 & \text{when } f(2a - x) = -f(x) \end{cases}$$

b) Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (x^3 - 1)^2$

50. a) Solve the following L.P.P.

Minimize and maximize $z = 3x + 9y$ subject to the constraints

$$x + 3y \leq 60$$

$$x + y \geq 10$$

$$x \leq y$$

$$x \geq 0, y \geq 0$$

b) Determine the value of k if $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$
